ELEMENTARY DYNAMICS

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National Council of Educational Research and Training

First Edition

Nov. 1990

P.D. 5 T

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Rs. 27 50

Published at the Publication Department by the Secretary, National Council of Educational Research and Training, NIE Campus, Sri Aurobindo Marg, New Delhi-110016 and Printed at Deep Printers, 3/26, Ramesh Nagar, New Delhi-15

Foreword

The present volume is the second of the two books which have been planned on elementary statics and dynamics. The first book on statics written by the same author had been published by NCERT in April, 1986. The two books together will cover all important topics of elementary Mechanics to meet primarily the needs of the Science students studying at the plus two stage. In some States, Mechanics has been included in the syllabus of Mathematics at the Senior Secondary stage. Students of these States as well as those appearing for many all-India competitive examinations may find these two books useful. Although Mechanics is studied in the books of physics, there is no scope of showing applications of various topics of mathematics in such books ; only the physical aspects are more emphasised. Students will find the application of various topics of mathematics such as trigonometry, algebra, geometry, mensuration, vector and calculus in these two volumes of Mechanics.

The present volume, like the previous one on elementary statics, begins with very elementary topics of dynamics and contains a lot of enrichment materials to satisfy the needs of the bright and gifted students. A large number of easy and difficult problems of various types have been worked out and included in the exercises to enable students to prepare for various examinations, competitive and others. The S.I. system of all physical quantities has been used throughout the book.

The necessity of producing two volumes of books on Mechanics for bright students has been felt for some time. NCERT, therefore, decided to undertake the production of these two volumes to meet the needs of Science students enrolled at the plus two stage.

The manuscripts prepared by Dr S.C. Das, Professor of Mathematics in NCERT's Department of Education in Science and Mathematics has been reviewed thoroughly by the subject specialists from Universities, Colleges & Schools all over the country in a national workshop organised by the Department. I am very glad to record my appreciation of the valuable contribution that Dr. Das has made through this book. I am also thankful to the subject specialists who were involved in the review of the manuscripts.

The council will appreciate suggestion which may help to further improve the contents and the presentation of the book.

P.L. MALHOTRA
Director
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Education Research &
Training

New Delhi

Preface

The present book is the second of my two books on Statics and Dynamics. The first book on Statics was already published by NCERT in 1986. This book like the previous one is mainly intended for the students of senior secondary classes for those States where it is included in the plus two syllabi of Mathematics. Students can also use this book for the preparation of various all-India Competitive Examinations where questions are asked in Mechanics. The need of studying Mechanics consisting of Statics and Dynamics in schools has been discussed in the "Preface" of the earlier book.

The topics chosen for this book are based on those which are followed by various States and various competitive examinations. Emphasis has been given on the basic theory as well as various types of challenging problems like that given in my previous book on Statics. Knowledge of mathematics beyond plus two stage has not been assumed anywhere in this book. A large number of problems, simple and difficult, have been worked out and given in the exercises so as to enable all students, gifted or average. to use the book without much difficulty. The S.I. System of all physical quantities has been used throughout this book. Some well known problems which were asked in various examinations have been included in this book with slight modification by changing the system of units and removing the vagueness in the traditional language. A number of multiple-choice objective type questions from each topic have been given in the appendix to facilitate students appearing at various all-India competitive examinations where this type of questions are generally asked. It is hoped that such students as well as students of physics may be benefited from this book.

The manuscripts, prepared by me, have been reviewed thoroughly by teachers of Schools, Colleges and Universities, who are specialised in the subject. They are Dr. A. Satyanarayan Rao, Andhra University, Prof. S. Basuli, Visva-Bharati, Shantiniketan, West Bengal, Dr. H.B.P. Sinha, Science College, Patna, Dr. M.K. Sen, North Bengal University, West Bengal, Prof. N.R. Chakraborty, A.C. College, Jalpaiguri, West Bengal, Dr. P.K. Mukherjee, Deshbandhu College, New Delhi, Dr. P. Biswas, P.D. Women's College, Jalpaiguri, Dr. P. Goswami, Deshbandhu College, New Delhi, Shri S. Shome, Fanindra Dev Institution, Jalpaiguri, West Bengal. Shri N. Lahiri, B.K.C. College, Calcutta, Shri S.S. Vashist

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Samajdar, Sonali Girls' School, Jalpaiguri, Shri D Saha, Jalpaiguri H S. School, Jalpaiguri, Miss. S. Majumdar, Sadar Girls' School, Jalpaiguri, Shri A. Banerjee, Adarsh Bidya Bhaban, Jalpaiguri and Shri M Guha, Kalna Maharaja's H.S. School, Kalna, Burdwan, West Bengal. They also developed the multiple-choice questions in the appendix, which were later revised by me. I am extremely grateful to all these teachers. I am also thankful to Shri D.D. Muhrlekar of Sardar Patel Vidyalaya, New Delhi, for his contribution in providing me with some good questions for this book. My thanks are also due to Shri A. Chakraborty of the Workshop Department of NCERT, New Delhi, who visualised the design of the cover of this book

Although maximum efforts have been made in detecting and removing the errors in the book, I shall be very grateful to any reader who may bring out errors, if any, to my notice. Suggestions from the readers for further improvement of the book will be highly appreciated.

S.C. Das
Author

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Chapter 1

Some Fundamental Concepts

1.1 Introduction

Dynamics is a branch of Mechanics which deals with the action of forces on bodies. Dynamics deals with the motion of bodies under the action of forces This is subdivided into two parts : (i) Kinematics and (ii) Kinetics.

Kinematics deals with the geometry of motion without any consideration of the causes of motion and *Kinetics* deals with the effect of the forces on the action of the body.

1 2 Some Definitions

The definitions of a rigid body, particle, mass etc. have been given in the author's book on "Elementary Statics". Here in this book, we give the definitions of some terms which will be used later in this book.

Motion of a body or a particle is the change of position of the body or the particle. When it is not changing its position, it is said to be at rest. A body has two kinds of motion: rotatory and translatory. When all the particles of a body have exactly the same change of position in the same direction, the body is said to perform a translatory motion. In a rotatory motion, a body moves round a fixed point or a fixed axis. When both translatory and rotatory motions are absent, the body is said to be at rest or in equilibrium. Sometimes a body may have both the types of motion simultaneously This type of motion has been discussed in the advanced books of Dynamics. In this elementary book, we shall be concerned with translatory motion of a particle only. Motion of a particle may be *uniform* or *variable* according to the manner of its movement with regard to time. The motion is said to be uniform when the particle passes over equal lengths of its path in equal intervals of time, however small. Otherwise, the the motion is said to be variable.

Speed of a particle is the rate at which it describes its path, straight or curved Speed does not give us any idea of direction. It is, therefore, a scalar quantity. Speed is uniform or variable according as equal or unequal lengths of path are described in any equal intervals, however small If a particle describes a distance S metres in time t seconds with uniform motion, then its speed is $\frac{S}{t}$ m/s In the case of variable speed, we require the *average speed* which is defined as the total *distance* divided by the total time.

The *displacement* of a moving particle during any interval of time is the change of its position, which is indicated by the straight line joining its initial position and the final position Displacement gives us the idea of both magnitude and direction and so it is a vector quantity

The *velocity* of a moving particle is the rate of change of its displacement. Thus, if V is the velocity and x is its displacement at any time t, then

$$V=\frac{dx}{dt}$$

The velocity has both magnitude and direction and so it is a vector quantity.

Remarks. (i) By "Velocity at an instant", we mean "the" average velocity during a very short time containing the instant under consideration. By an *average velocity*, we mean the ratio of the *displacement* undergone to the time taken Thus, if δx be the small displacement described in small time δt, then the average velocity$=\frac{\delta x}{\delta t}$and

$$V=\lim_{\delta t\to 0}\frac{\delta x}{\delta t}=\frac{dx}{dt}$$

(ii) Strictly speaking, velocity should be denoted by $\vec{V}$ instead of simply V, since velocity is a vector quantity. But conventionally, we generally say that a particle has a velocity V. What we understand by this statement is that the particle has a velocity of *magnitude* V i.e. V is the speed in a definite direction.

Acceleration is the rate of change of velocity. Thus, if V is the velocity at any time t and the displacement is x during this time, then acceleration $f=\frac{dV}{dt}=\frac{d^2x}{dt^2}$

Thus, acceleration is also a vector quantity The above formula gives only the magnitude of the acceleration A negative acceleration is called *retardation*

1.3 A Remark on the Traditional Nomenclature of Physical Quantities

We know that velocity, acceleration, force, displacement, momentum, etc. are all vector quantities, Hence, the following statements which are conventionally used in books of Physics and Mechanics are strictly speaking wrong.

(i) The velocity of a particle is 10m/s.
(ii) A particle may have several displacements, say 15m, 8m, 3m in different directions.
(iii) A force of 10 N acts at a point.
(iv) The momentum of a body is 25 gramme-metre.

It is clear that vectors cannot be represented by numbers only. What we mean by the above statements is simply the magnitudes of the corresponding vector quantities Thus when we say that the velocity of a particle is 10m/s, we mean that the velocity of the particle is *of magnitude* 10m/s

Similarly, other statements can be explained We shall very often use the same traditional language in this book but the actual meaning of the statements should be understood as explaided above.

1.4 Solved Examples

Ex. 1. Find the average speed of a bicycle over a journey of 50 km if it travels the first 10 km at 20 km/h, the second 12 km in 1h, the third 24 km at 8 km/h and the last 4 km at 8 km/h.

Time taken by the bicycle to travel the first 10 km$=\frac{10}{20}$ h $\frac{1}{2}$ h.

Similarly, the times taken by the bicycle to travel the second 12 km, the third 24 km and the last 4 km are respectively 1h, $\frac{24}{8}$h and $\frac{4}{8}$h i.e. 1h, 3h and $\frac{1}{2}$ h.

Hence, the total time for the whole journey

$\left(\frac{1}{2}+1+3+\frac{1}{2}\right)h=5$h.

Also, the total distance$=50$ km.

Hence, the average speed$=\dfrac{\text{Distance}}{\text{Time}}$

$$=\frac{50}{5}\text{ km/h}=10\text{ km/h}$$

EXERCISE 1

1. A man walks a distance of 3 km towards the north-east at the rate of 6 km/h and then walks a distance of 4 km towards the south-east at the rate of 2 km/h. Find the average speed of the man for the whole journey. [Ans. 2.8 km/h]

2. A point moves in a straight line with a velocity of 3 m/s. After 3s, it has an additional velocity of 4 m/s at right angles to its original direction of motion. Find its distance from the starting point 2 s after this. [Ans. 17 m]

3 A particle moves towards east from A to B at the rate of 4 km/h and then towards north from B to C at the rate of 5 km/h Finally it reaches A. If $AB=12$ km and $BC=10$ km, find the average velocity of the particle for the whole journey. [Ans. 0 km/h]

Chapter II

Composition and Resolution of Velocities

2.1 Resultant and Components

If a particle possesses several simultaneous velocities in different directions such that the joint effect of all the velocities is the same as if the particle moves with a single velocity in a definite direction, then this single velocity is called the *resultant* of the given velocities. The given velocities are known as the *components* of the single resultant.

2.2 Parallelogram of Velocities

We know that if two vectors are represented in magnitude and direction by two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn through the point of intersection of the two vectors. Since the velocity is a vector quantity, we can also find the resultant of two velocities possessed by a particle in a similar way. The law is generally known as *Parallelogram of Velocities*, which can be stated as follows:

If a particle has two simultaneous velocities represented in magnitude and direction by two adjacent sides of a parallelogram drawn from an angular point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from that point.

Some examples of particles or bodies possessing two or more simultaneous velocities can be given. Suppose a boy is walking on the floor of a running train with a certain velocity. Clearly, the boy possesses two velocities: his own velocity and the velocity of the train. A small insect moving on a boat which is again driven by a man in a definite direction in a river flowing with certain velocity in another direction has three velocities: (i) its own velocity (ii) the velocity of the boat and (iii) the velocity of the current

2.3 Analytical Expression for the Resultant of Two Given Velocities

Let a particle at O have two velocities $\vec{u}$ and $\vec{v}$ represented by $\vec{OA}$ and $\vec{OC}$ respectively. We complete the parallelogram $OABC$ and join the diagonal OB. By parallelogram of velocities, $\vec{OB}$ represents the resultant velocity, say $\vec{w}$, of $\vec{u}$ and $\vec{v}$

Let $\angle COA=\alpha$ and $\angle BOA=\theta$

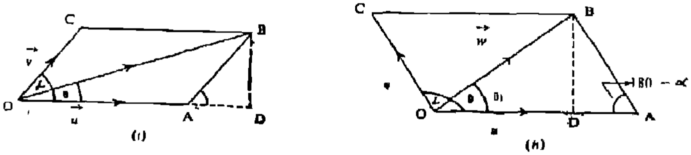


Fig 1

We draw BD perpendicular to OA [fig. 1 (ii)] or OA produced [fig. 1 (i)]. Clearly, $OA=u$, $AB=OC=v$ and $OB=w$. $\angle BAD=\alpha$ in fig. 1 (i) and $\angle BAD=180°-\alpha$ in fig. 1 (ii).

Now, in fig. 1 (i), we have from the right-angled triangle OBD,

$$OB^2=OD^2+BD^2$$

$$\Rightarrow w^2=(OA+AD)^2+BD^2$$

$$=(u+v\cos\alpha)^2+v^2\sin^2\alpha \qquad [\because AD=v\cos\alpha \text{ and } BD=v\sin\alpha \text{ from right-angled triangle } ABD]$$

$$=u^2+2uv\cos\alpha+v^2\cos^2\alpha+v^2\sin^2\alpha$$

$$=u^2+2uv\cos\alpha+v^2(\cos^2\alpha+\sin^2\alpha)$$

$$=u^2+2uv\cos\alpha+v^2.$$

Hence, $w=\sqrt{u^2+2uv\cos\alpha+v^2}$

and $\tan\theta=\dfrac{BD}{OD}=\dfrac{v\sin\alpha}{u+v\cos\alpha}$

In fig. 1 (ii), we have from the right-angled triangle OBD,

$$OB^2=OD^2+BD^2$$

$$\Rightarrow w^2=(OA-AD)^2+BD^2$$

$$=\{u-v\cos(180^\circ-\alpha)\}^2+v^2\sin^2(180^\circ-\alpha)$$

[$\because$ From right-angled triangle ABD, $AD=v\cos(180^\circ-\alpha)$ and $BD=v\sin(180^\circ-\alpha)$]

$$=(u+v\cos\alpha)^2+v^2\sin^2\alpha$$

$$=u^2+2uv\cos\alpha+v^2(\cos^2\alpha+\sin^2\alpha)$$

$$=u^2+2uv\cos\alpha+v^2$$

and $$\tan\theta=\frac{BD}{OD}=\frac{v\sin(180^\circ-\alpha)}{u-v\cos(180^\circ-\alpha)}=\frac{v\sin\alpha}{u+v\cos\alpha}.$$

Hence, in both the cases, we see that *the resultant $\vec{w}$ of two velocities $\vec{u}$ and $\vec{v}$ is given by the formula*

$$w=\sqrt{u^2+v^2+2uv\cos\alpha}$$

and the direction of the resultant is given by

$$\theta=\tan^{-1}\left(\frac{u\sin\alpha}{u+v\cos\alpha}\right)$$

where θ is the angle made by the resultant with the component velocity u.

Particular Cases :

(*i*) If $\alpha=0$, then $w=u+v$ In this case w is maximum. Hence, *the resultant of two simultaneous velocities along the same line in the same direction is their sum. Also, the resultant of two velocities is maximum only when they act along the same line and in the same direction.*

(*ii*) If $\alpha=\pi$, then $w=u-v$ if $u>v$

$=v-u$ if $v>u$

In this case, w is minimum. Hence, *the resultant of two simultaneous velocities acting along the same line but in the opposite directions is the difference of the component velocities and acts in the direction of the greater velocity. Also the resultant of two velocities is minimum only when they act along the same line but in the opposite directions.*

(*iii*) If $\alpha=\frac{\pi}{2}$, then $w=\sqrt{u^2+v^2}$.

Hence, the resultant of two simultaneous velocities, $\vec{u}$ and $\vec{v}$, acting at right angles to each other is of magnitude $\sqrt{u^2+v^2}$.

(*iv*) When the magnitudes of two component velocities are equal *i.e.* when $v=u$, then,

$$w=\sqrt{2u^2+2u^2\cos\alpha}$$

$$=\sqrt{2}\,u\sqrt{2\cos^2\frac{\alpha}{2}}=2u\cos\frac{\alpha}{2}.$$

In this case

$$\tan\theta=\frac{u\sin\alpha}{u\,(1+\cos\alpha)}=\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}$$

$$=\tan\frac{\alpha}{2}$$

so that $\theta=\frac{\alpha}{2}$.

Hence, *the resultant of two equal velocities each of magnitude u acting at an angle α with each other is $2u\cos\frac{\alpha}{2}$ and acts in a direction bisecting the angle between them.*

2.4 Component Velocities in Given Directions

A given velocity can be resolved into two components in different directions in an infinite number of ways, for we can construct an infinite number of parallelograms with the straight line representing the given velocity as a diagonal and the two adjacent sides of each of these parallelograms will represent the two component velocities But we can find the components of a given velocity in two given directions in a unique way.

Let OB represent the given velocity $\vec{v}$ and let OX and OY be two given directions making angles α and β to the line OB as shown in fig. 2

We shall find the components of $\vec{v}$ along OX and OY.

With OB as a diagonal, we construct a parallelogram $OABC$. Then OA and OC represent the magnitudes of the components of the given velocity $\vec{v}$.

Now in $\Delta\,OAB$, we have

$\angle AOB=\alpha$, $\angle OBA=\angle BOC=\beta$ and $\angle OAB=180^\circ-(\alpha+\beta)$

Hence, by using sine rule in $\Delta\,OAB$, we get

$$\frac{OA}{\sin\angle OBA}=\frac{AB}{\sin\angle AOB}=\frac{OB}{\sin\angle OAB}$$

$$\Rightarrow\frac{OA}{\sin\beta}=\frac{AB}{\sin\alpha}=\frac{v}{\sin[180^\circ-(\alpha+\beta)]}=\frac{v}{\sin(\alpha+\beta)}$$

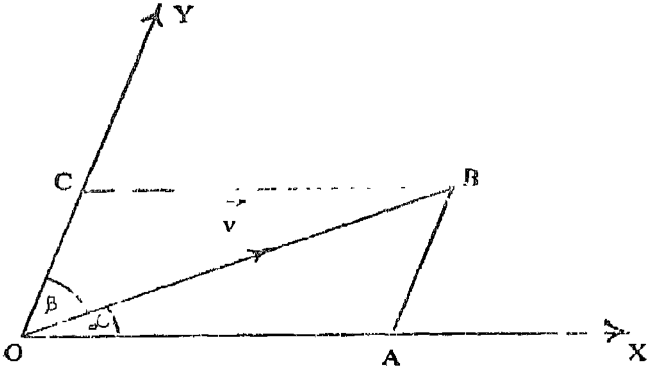


Fig. 2

Hence, $$OA=\frac{v\sin\beta}{\sin(\alpha+\beta)} \qquad (i)$$

and $$AB=\frac{v\sin\alpha}{\sin(\alpha+\beta)}$$

But $$AB=OC$$

$$\therefore \quad OC=\frac{v\sin\alpha}{\sin(\alpha+\beta)}. \qquad (ii)$$

Thus the required components of $\vec{v}$ are given by (*i*) and (*ii*).

2.5 Components of a Velocity in Two Perpendicular Directions

When the directions of the components of a given velocity are at right angles to one another, then the components are generally called *resolved parts* in the corresponding directions.

Let *OB* represent the given velocity $\vec{v}$ and let *OX* and *OY* be two perpendicular lines through *O*. We shall resolve the velocity $\vec{v}$ along *OA* and *OC*. Let $\vec{v}$ make an angle θ with *OX*, where θ may be acute (as in fig. 3 (*i*)) or obtuse (as in fig. 3 (ii)). With *OB* as a diagonal, we construct a rectangle *OABC*. Hence, by parallelogram of vectors, *OA* and *OC* represent the magnitude of the components of $\vec{v}$.

In fig. 3 (i), we have from right-angled triangle OAB,

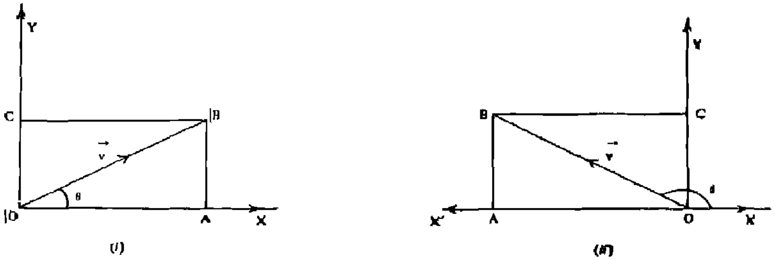
$$OA = v\cos\theta$$

and $$OC = AB = v\sin\theta$$

In fig. 3 (ii),

$$OA = v\cos(180^\circ - \theta) = -v\cos\theta$$

and $$OC = v\sin\theta$$



(i) (ii)

Fig. 3

Now, $-v\cos\theta$ along OX' is the same as $v\cos\theta$ along OX. Hence, *the resolved parts of any velocity v acting at O making an angle θ with OX are $v\cos\theta$ along OX and $v\sin\theta$ along a perpendicular direction OY whatever the angle θ may be.*

Remark : It follows from the above discussion that any given velocity v can always be replaced by two simultaneous resolved parts, $v\cos\theta$ along a direction OX making an angle θ to the direction of v and $v\sin\theta$ perpendicular to OX.

2 6 Resultant of a Number of Coplanar Velocities of a Particle

Let a particle at O have a number of coplanar velocities $v_1, v_2, \ldots v_n$ making angles $\theta_1, \theta_2 \ldots \theta_n$ respectively with OX in the plane.

Let $OY \perp OX$. We can replace the velocity v_1 by its components $v_1\cos\theta_1$ along OX and $v_1\sin\theta_1$ along OY Similarly, we can replace each of the other velocities by two components along OX and OY. Then the given simultaneous velocities are equivalent to $v_1\cos\theta_1, v_2\cos\theta_2, \ldots v_n\cos\theta_n$ along OX and $v_1\sin\theta_1, v_2\sin\theta_2, \ldots v_n\sin\theta_n$ along OY. All the velocities along OX can be combined to a single velocity $\sum_{r=1}^{n} v_r\cos\theta_r$ and those along OY can be combined to a single velocity $\sum_{r=1}^{n} v_r\sin\theta_r$.

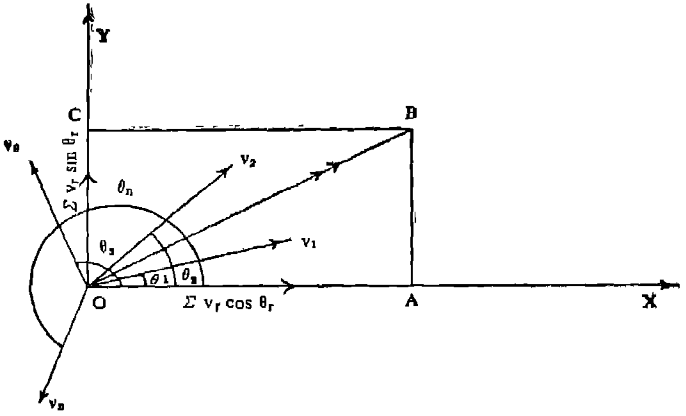


Fig. 4

If we represent these two final components by OA and OC respectively (fig. 4), then their resultant velocity V will be represented by the diagonal OB of the rectangle $OABC$. Let OB make an angle θ with OX.

$$V\cos\theta = \sum_{r=1}^{n} v_r \cos\theta_r \qquad ..(i)$$

and

$$V\sin\theta = \sum_{r=1}^{n} v_r \sin\theta_r. \qquad ...(ii)$$

Squaring (i) and (ii) and adding, we get

$$V = \sqrt{\left(\sum_{r=1}^{n} v_r \cos\theta_r\right)^2 + \left(\sum_{r=1}^{n} v_r \sin\theta_r\right)^2} \qquad ...(iii)$$

Dividing (ii) by (i), we get

$$\tan\theta = \frac{\sum_{r=1}^{n} v_r \sin\theta_r}{\sum_{r=1}^{n} v_r \cos\theta_r} \qquad \cdot (iv)$$

Hence, the magnitude and the direction of the resultant velocity are given by (iii) and (iv).

Remark : From (i) and (ii) above it follows that *the algebraic sum of the resolved parts of any number of velocities in a given direction is equal to the resolved part of their resultant in the same direction.*

2.7 Polygon of velocities

If a moving point possesses simultaneous velocities represented by the sides AB, BC, CD, . . . ZA of any polygon in magnitude and direction, then their resultant velocity is represented by AZ which closes the polygon, in magnitude and direction.

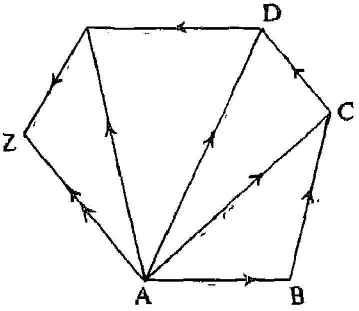


Fig. 5

This follows from triangle of vectors or velocities. For this, we join *AC*, *AD*,. . and form triangles *ABC*, *ACD*, ...

Now, by triangle of velocities, we have

$$\vec{AB}+\vec{BC}=\vec{AC}$$

$$\vec{AC}+\vec{CD}=\vec{AD}$$

.. ..

....

and so on.

Hence, the resultant of velocities *AB*, *BC*, *CD*.. is given by *AZ*.

Remark : It follows from the above discussion that if a particle possesses simultaneous velocities represented in magnitude and direction by the sides of a closed polygon, taken in order, then the particle will remain at rest

2.8 Solved Examples

Ex. 1. A particle having simultaneous velocities 3m/s, 5m/s and 7m/s is at rest. Find the angle between the first two velocities.

Let θ be the angle between the velocities of 3 m/s and 5 m/s Since the three simultaneous velocities are at rest, the resultant of 3 m/s and 5 m/s must be equal to 7 m/s in magnitude and opposite in sign

Hence, we have

$$7^2=3^2+5^2+2\times3\times5\cos\theta$$

$$\Rightarrow 49=9+25+30\cos\theta$$

$$\Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2}=\cos 60^\circ.$$

Hence, $\theta=60^\circ$ i.e. the angle between the first two velocities is 60°.

Ex. 2 A man rows directly across a flowing river in time t_1 and rows an equal distance down the stream in time t_2. If u be the speed of the man in still water and v be that of the stream, show that

$$t_1 : t_2=\sqrt{u+v} : \sqrt{u-v}.$$

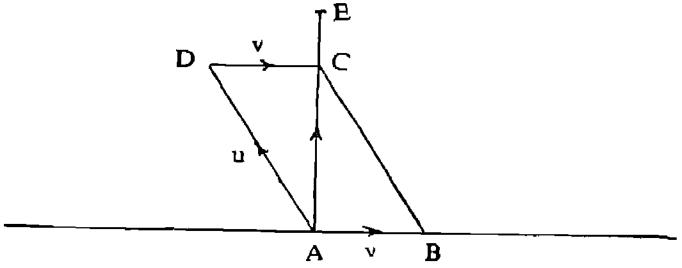


Fig. 6

Let AB represent the velocity of the stream and AD represent that of the man. The resultant velocity AC which is the diagonal of the parallelogram $ABCD$ is perpendicular to the bank AB. Now, $AC=u^2-v^2$ (from right-angled ΔADC)

If the distance $AE=S$ is travelled by the man in time t_1, then

$$t_1=\frac{S}{\sqrt{u^2-v^2}} \qquad \text{...(i)}$$

If the man rows an equal distance S down the stream, then the resultant velocity of the man and the stream is $u+v$ Hence, if t_2 be the time, then

$$t_2=\frac{S}{u+v} \qquad \text{...(ii)}$$

Dividing (i) by (ii), we get

$$\frac{t_1}{t_2}=\frac{S}{\sqrt{u+v}\,\sqrt{u-v}}\times\frac{u+v}{S}=\frac{\sqrt{u+v}}{\sqrt{u-v}}.$$

Hence, $t_1 . t_2=\sqrt{u+v} : \sqrt{u-v}$.

Ex 3. Two trains, each 210 m long, are moving towards each other on parallel lines with velocities 36 km/h and 48 km/h respectively. Find the time that elapses from the instant when they first meet until they have cleared each other.

We know that

36 km/h=36000 m/h and

48 km/h=48000 m/h.

In 1 hour, the two trains describe a distance of (36000+48000) m =84000 m.

When the two trains clear each other from the instant when they first meet, they will describe a distance equal to the sum of their lengths i e. 420 m. Hence, the required time

$$=\frac{420\times60\times60}{84000}\text{ s}=18\text{s}$$

Ex. 4. A destroyer steaming north at the rate of 24 km per hour observes a sea-plane carrier due east of itself at a distance of 16 km, the latter steaming due west at the rate of 32 km per hour. After what time are they at the least distance from one another and what is this least distance ?

Let the destroyer be moving due north in the direction *OB* and the sea-plane be moving due west starting from *A* due east of *O*. Let after time *t* hours, *A'* be the position of the sea-plane In time *t* hours, the sea-plane traverses a distance of 32 *t* km and the destroyer moves a distance of 24 *t* km so that *AA'*=32 *t* km, and *OB*=24 *t* km

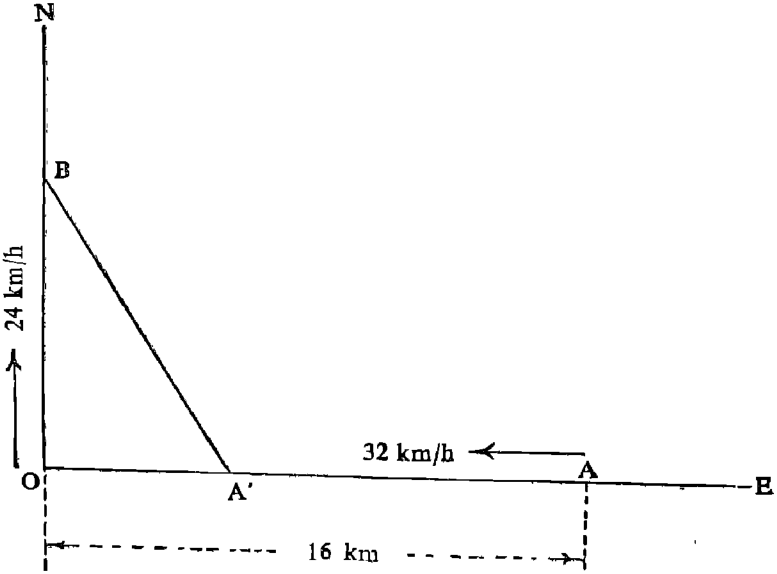


Fig. 7

$\therefore\ OA' = (16-32t)$ km.

Now, since OA is towards east and OB is towards north, hence, $\angle BOA = 90^\circ$

$$\therefore\ A'B^2 = OB^2 + OA'^2$$
$$= (24t)^2 + (16-32t)^2.$$

If the distance between the sea-plane and the destroyer after t hours be S, then

$$S^2 = 24^2 t^2 + (16-32\ t)^2$$
$$= 576\ t^2 + 256\ (1-2\ t)^2$$

We shall find the minimum value of S^2.

Now, $\dfrac{dS^2}{dt} = 1152\ t - 2\times 2\times 256\ (1-2t)$

For maximum or minimum,

$$\frac{dS^2}{dt} = 0$$

$\Rightarrow\quad 1152\ t - 1024 + 2048\ t = 0$

$\Rightarrow\quad 3200\ t = 1024$

$\Rightarrow\quad t = \dfrac{1024}{3200} = \dfrac{8}{25}$

Now, $\dfrac{d^2S^2}{dt^2} = 1152 + 2048 = 3200$

which is positive for all values of t.

Hence, S^2 is minimum for $t = \dfrac{8}{25}$

i.e. $\quad S$ is minimum for $t = \dfrac{8}{25}$.

Hence, the minimum distance is

$$\sqrt{576\times\left(\frac{8}{25}\right)^2 + 256\left(1-\frac{16}{25}\right)^2}\ \text{km} = \sqrt{\frac{94464}{625}}\ \text{km}$$
$$= \frac{307.35}{25}\ \text{km} = 12.29\ \text{km}.$$

Hence, the distance between the destroyer and the sea-plane will be least after $\dfrac{8}{25}$ h $= \dfrac{8\times 60}{25}$ min $= 19.2$ min and the least distance is 12.29 km

Ex. 5 A man can swim directly across a stream of width S metres in t min when there is no current and in t' min when there is current. Show that the velocity of the current is

$$S.\sqrt{\frac{1}{t^2}-\frac{1}{t'^2}}\ \text{m/min.} \qquad \text{[C.U. 1954]}$$

Let u m/min and v m/min be the velocities of the man and the current respectively. Clearly, $u=\dfrac{S}{t}$.

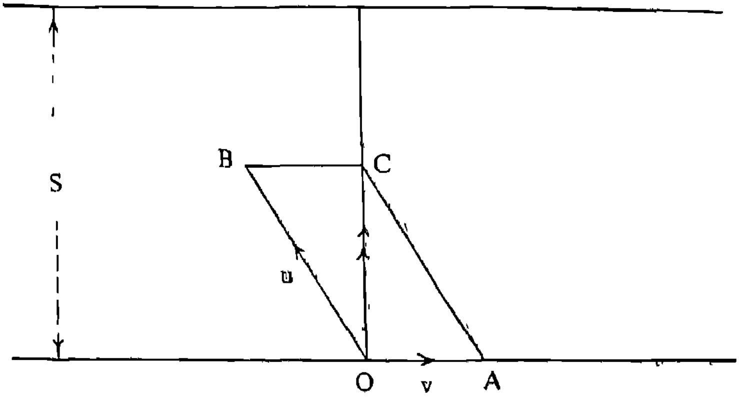


Fig. 8

Let OA represent the velocity of the current and OB represent the velocity of the man so that $OA=v$ and $OB=u$. When there is a current, the man must swim in the direction such that the direction of the resultant velocity OC is perpendicular to the bank OA of the river. Since OC is the resultant of u and v, hence OC must be the diagonal of the parallelogram $OACB$ Now, from the figure,

$$OC=\sqrt{OB^2-BC^2}=\sqrt{u^2-v^2}=\sqrt{\frac{S^2}{t^2}-v^2}.$$

If the man takes t' min to cross the river of breadth S metres with this resultant velocity, then

$$t'=\frac{S}{\sqrt{\frac{S^2}{t^2}-v^2}}$$

$$\Rightarrow \quad \frac{S^2}{t'^2}=\frac{S^2}{t^2}-v^2$$

$$\Rightarrow \quad v^2=S^2\left(\frac{1}{t^2}-\frac{1}{t'^2}\right)$$

$$\Rightarrow \quad v=S\sqrt{\frac{1}{t^2}-\frac{t}{t^2}}\,.$$

Hence the result

Ex. 6. Two boats each moving with a velocity of 5 km/h try to cross a stream of breadth $\frac{1}{2}$ km running with a velocity of 3 km/h. One boat crosses the stream by the shortest path and the other in the shortest time If they start together, find the interval between their times of arriving at the opposite bank.

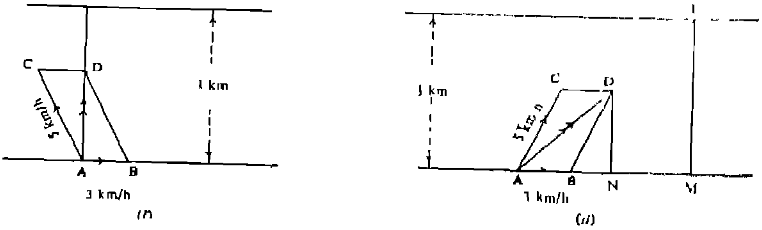


Fig. 9.

In fig 9, let AB represent the velocity 3 km/h of the current and AC represent the velocity 5 km/h of the boat.

The resultant velocity of the boat due to these two simultaneous velocities will be represented by the diagonal AD of the parallelogram $ABCD$ (figs. (i) and (ii)).

When a boat crosses the river by the shortest path, the resultant velocity must be perpendicular to the bank so that AD is perpendicular to AB (fig (i)). If t_1 h be the time of arrival of this boat on the opposite bank, then

$$t_1=\frac{\text{breadth of the river}}{\text{resultant velocity}}$$

$$=\frac{\frac{1}{2}}{AD}\text{h}=\frac{1}{2\sqrt{AC^2-CD^2}}$$

$$=\frac{1}{2\sqrt{5^2-3^2}}=\frac{1}{8}\text{h}.$$

Again, if the second boat wishes to cross the river in the shortest time, let the direction of the boat make an angle α with the bank (fig. (ii)). In unit time, the boat will cross a distance equal to AD. We produce AD to meet the opposite bank at E. Let $EM \perp AB$ and $DN \perp AB$.

If t_2 h be the time to cross the river, then

$$t_2=\frac{AE}{AD}=\frac{EM}{DN}$$

$$=\frac{\frac{1}{2}}{DB\sin\alpha}=\frac{1}{2\times 5\sin\alpha}=\frac{1}{10\sin\alpha}$$ (from similar triangles ADN and EAM)

Now, t_2 will be minimum when $\sin\alpha$ is maximum i.e when $\alpha=\frac{\pi}{2}$.

Hence, the minimum value of t_2

$$=\frac{1}{10\sin\frac{\pi}{2}}=\frac{1}{10}.$$

Hence, the boat will reach the opposite bank in the shortest time of $\frac{1}{10}$ h. The required interval between these two times is, therefore,

$$\left(\frac{1}{8}-\frac{1}{10}\right)\text{h}=\frac{1}{40}\text{ h}.$$

EXERCISE 2

1 If u and v be the components of the resultant velocity w, α be the angle between u and v and θ be the angle between the directions of v and w, find

(i) α if $w=10$, $u=8$, $v=6$ [Ans. $\alpha=90°$]

(ii) u, if $v=60$, $w=30\sqrt{7}$, $\alpha=60°$ [Ans. $u=30$]

(iii) θ, if $u=2$, $v=4$ and $w=2\sqrt{3}$ [Ans. $\theta=90°$]

(iv) α and θ, if $u=v=w$ [Ans. $\alpha=120°$, $\theta=60°$]

(v) α and θ, if $u=v=\frac{1}{2}w$ [Ans. $\alpha=\theta=0°$]

(vi) $u : v \cdot w$, if $\alpha=90°$ and $\theta=45°$ [Ans. $u=v$ and $w=\sqrt{2}\,u$]

2 A thief, when detected, jumps out of a running train at right angles to its direction with a velocity of 5m/min If the velocity of the train is 36 km/h, find in which direction the thief falls.

[Ans. At an angle $\tan^{-1}\frac{1}{120}$ with the direction of motion of the train]

3. A box is sliding down a slope inclined at an angle of 30° at a constant velocity of 1m/s. Find the vertical and the horizontal components of the velocity

[Ans. $\frac{1}{2}$ m/s and $\frac{\sqrt{3}}{2}$ m/s]

4 The escalator at a sub-way station moves at a velocity of 8 m/s. Find the depth of the tunnel if the angle made by the escalator with the horizontal equals 30° and it brings a man to the surface in 2.5 min [Ans. 600 m]

5. A railway carriage is travelling at 20 m/s and a person rolls a ball across the floor of the carriage at right angles to the direction of motion of the train at 15 m/s Find the resultant velocity of the ball.

[Ans. 25 m/s an angle $\tan^{-1}\frac{3}{4}$ with the direction of motion of the train]

6. The resultant of two velocities is 20 m/s at right angles to the component which is 15 m/s. Find the magnitude and direction of the other component

[Ans. 25 m/s in a direction making an angle $\tan^{-1}\frac{3}{4}$ with the resultant]

7. An aeroplane is ascending at an angle of 30° to the horizon at a velocity of 216 km/h What altitude will the plane reach in 10s ? [Ans. 300 m]

8. The speed of a boat in a river is u m/s and that of the current is v m/s. The boat traverses a distance of d m downstream and then comes back to its original position. Find the average speed of the boat for to and fro journey.

[Ans. $\frac{u^2-v^2}{u}$ m/s]

9. A plane which can complete a course in the form of a square in time 4 t at a constant speed v when there is no wind, flies over the same course with a wind of velocity kv (where $k<1$),

blowing parallel to one side, prove that the time will be $2t\left(\frac{2-k^2}{1-k^2}\right)$ and find the time taken if the wind with velocity kv had been blowing parallel to a diagonal of the square.

$$\left[\text{Ans. } \frac{8t}{2-k^2}\right]$$

10. A particle has three velocities 10 m/s, 20 m/s and 30 m/s inclined at angles of 120° to one another. Find the resultant velocity.

[Ans $10\sqrt{3}$ m/s in a direction at right angles to the second velocity]

11 A point possesses simultaneous velocities of 8 m/s, 5 m/s, 10 m/s and 10 m/s towards north, south, east and west respectively. Determine the magnitude and direction of the resultant.

[Ans. 3 m/s towards north]

12. If a point has a velocity of 1 m/s to the east and also a velocity of $\sqrt{3}$ m/s to the north, determine the velocity which must be compounded with these to bring the point to rest.

[Ans 2 m/s making an angle of 60° with the east]

13. A particle possessing velocities 3 m/s, 7 m/s and 8 m/s is at rest. Find the angle between the greatest and the least of the velocities.

[Ans. 120°]

14. A point possesses four simultaneous velocities whose magnitudes are 4 m/s, 8 m/s, $12\sqrt{3}$ m/s and 16 m/s respectively. The angle between the directions of the first and the second is 60°, between the second and the third 90°, and between the third and the fourth 150°. Find the magnitude and direction of the resultant velocity.

[Ans. 4 m/s in a direction making an angle 120° with the first velocity]

15. Three velocities whose ratios are $1 : 2 : \sqrt{3}$ are simultaneously impressed on a particle and it is found that the particle is at rest. Determine the angles at which the directions of the velocities are inclined to each other.

[Ans. Angles between 2nd and 3rd velocities, 3rd and 1st, and 1st and 2nd are respectively 90°, 150° and 120°]

16. A man walks at 10 m/s in the eastern direction across the deck of a ship which is sailing $20\sqrt{2}$ m/s in north-east direction. There is a tide in the direction 30° east of north running at

30 m/s The wind is blowing at the rate of $15\sqrt{3}$ m/s in the southern direction. Find the velocity of the man in space.

[Ans. 49.24 m/s at $\tan^{-1}\frac{9}{4}$ in east of north]

17 A ball is moving horizontally with a velocity of 90 m/s. What additional velocity must be given to this ball so as to make it travel at right angles to its original path with a velocity of 120 m/s ?

[Ans 150 m/s at an angle $\cos^{-1}\left(-\frac{3}{5}\right)$ with the original path]

18. How far down the stream will a swimmer reach the opposite bank in a stream of width 500 m if he swims at right angles to the stream with twice the velocity of the current?

[Ans. 150 m

19. A point has equal velocities in two given directions. If one o these velocities be halved, the angle which the resultant make with the other is also halved Show that the angle between th velocities is 120°.

20. A train is moving with a uniform velocity v along a straigh railway line and a motor car runs on a parallel road in the sam direction, the distance between the road and the railway lin being a. A passenger of the train observes the car to be alway in a line with a fixed tree whose distance from the railway line i b $(b>a)$. Prove that the velocity of the car is uniform and fin its magnitude.

[Ans. $\left(1 \quad \frac{a}{b}\right)$

21 A particle P has simultaneously three velocitie represented b $\overrightarrow{PA}$, $\overrightarrow{PB}$, $\overrightarrow{PC}$, where A, B, C are three non-collinear fixed point Where should P be situated so that the particle may remain a rest ?

[Ans. At the centroid of ΔABC

22. A swimmer can swim in still water at the rate of 4 km/h. H wishes to cross a river flowing along a straight course at the ra of 2 km/h so as to reach directly the opposite point on the oth bank. In what direction should he attempt to swim?

If he wishes to cross the river in the shortest time, what direction should he take to swim?

[Ans In the first case, the swimmer should swim at an angle 120° with the direction of the current. In the second case the swimmer should swim in a direction perpendicular to the current]

23. A man who swims at the rate of 5 m/s in still water wishes to cross a river flowing at the rate of 3 m/s Find the direction in which he must swim in order to go straight across the stream. If the breadth of the stream be 200 m, find the time taken in crossing the river

[Ans At at angle $\cos^{-1}\left(-\frac{3}{5}\right)$ with the direction of the current, 50 s]

24. Two motor buses, each of breadth b, move one behind the other with velocity v along a narrow street and there is a clear space of length s between them. Show that the least velocity with which a pedestrian could go across between them is $\frac{bv}{\sqrt{s^2+b^2}}$ and find in what direction he should cross.

[Ans At $\tan^{-1}\frac{s}{b}$ with the breadth of each bus]

25 A vessel steams with the bow pointed due north with a velocity of 18 km/h and is carried by a current which flows in south easterly direction at the rate of $10\sqrt{2}$ km/h. At the end of an hour, find its distance and bearing from the point from where it started.

[Ans. 12.8 km, $\tan^{-1}\frac{5}{4}$ due east of north]

26. A ship is steaming in a direction due north across a current running due west At the end of one hour, it is found that the ship has made $8\sqrt{3}$ km in a direction 30° west of north Find the velocity of the current and the rate at which the ship is steaming.

[Ans. $4\sqrt{3}$ km/h and 12 km/h]

27 A man wishes to go straight across a river. If he can row his boat with three times the velocity of the current, find at what inclination to the current he must keep the boat pointed?

$\left[\text{Ans. } \cos^{-1}\left(-\frac{1}{3}\right)\right]$

28 A river of breadth 1 km has a current flowing at the rate of 2 km/h A swimmer who can swim at the rate of 4 km/h in still water wishes to reach the directly opposite point on the other bank, but choosing a wrong direction to swim, he reaches the opposite bank $\frac{1}{2}$ km down the desired point. Find the deviation of the chosen direction from the right one

[Ans. 30°]

29 A boat X is sailing with a velocity u due north and another Y, with a velocity v due west. If initially X were at a distance x west of Y, find the time after which they will be nearest to each other and also their shortest distance.

$$\left[\text{Ans. Time} = \frac{xv}{u^2+v^2}\right.$$

$$\left.\text{and shortest distance} = \frac{xu}{\sqrt{u^2+v^2}}\right]$$

30 Two straight railway lines meet at right angles A train starts from the junction along one line, and at the same instant, another train starts towards the junction from a station on the other line, and they move at the same uniform speed. Show that they are nearest to each other when they are equally distant from the junction.

31. A man swimming in a stream of velocity v suddenly finds himself to be distant d from a fall of water ahead. If c be the distance of the nearest shore, show that to save himself, the man must swim with a velocity not less than $\frac{cv}{\sqrt{c^2+d^2}}$.

32. Two motor boats start simultaneously from two points, A and B, the first one moving with a uniform velocity of $10\sqrt{3}$ km/h in a direction making an angle of 30° with AB Find the direction which the second should move uniformly with a velocity of 10 km/h so that it may meet the first one. If the second boat moves at an angle of 45° with AB, with what velocity should it go in order that it may meet the first?

[Ans. 60° with BA, $5\sqrt{6}$ km/h]

33 Two particles, A and B, are moving in two straight lines which meet at right angles at C, A approaching C, and B receding from it Prove that if they are always at the same distance apart, the ratio of the velocity of A to that of B at each instant will be equal to the ratio of CB to CA.

34. Cyclists P and Q are respectively at points A and B which are $(\sqrt{3}+1)$ km apart on a field. P rides away with a uniform

velocity of $5\sqrt{2}$ km/h in a direction making an angle 45° with AB. Q starts at the same instance to move with a uniform velocity of 10 km/h and catches P. Find the time that elapses from start before Q catches P.

[Ans. 12 min]

35 An aeroplane flies daily a straight line course from A to B and back. In calm weather, the speed is u and the time for the double journey is T. On a certain day, the velocity of the wind is v and in a direction inclined at θ to AB Prove that on the onward and the backward journeys, the aeroplane has to be headed in directions equally inclined to AB, and the inclination to it is $\sin^{-1}\left(\frac{v \sin \theta}{u}\right)$; prove also that the time for the double journey is $Tu\sqrt{\frac{u^2-v^2 \sin^2 \theta}{u^2-v^2}}$.

36. An aeroplane, travelling in still air at the rate of p km/h starts from a point P to reach a point O due north of it $2d$ km away There is a wind blowing due west at the rate of q km/h, but when half the distance has been covered, the velocity of the wind increases to r km/h ($p>r>q$) and the aeroplane adjusts its head accordingly so that it continues its course along PQ as before, and reaches Q. Show that the time taken over the flight is

$$d\left(\frac{1}{\sqrt{p^2-q^2}}+\frac{1}{\sqrt{p^2-r^2}}\right) \text{ hours}$$

37. A battleship leaves a certain port and steams due north-west at a km/h. Another ship leaves the same port at the same instant and steams due $22\frac{1}{2}°$ south of west at b km/h. Their wireless instruments are capable of communication upto c km. Show that the ships are expected to remain in touch with each other for $\frac{c}{\sqrt{a^2+b^2-ab\sqrt{2-\sqrt{2}}}}$ hours.

38. A rowing-boat which can be propelled at 3 km/h in still water is caught in a current moving at 4 km/h to the north-west The rower wishes to reach a second point 6 km to the north. Show that there are two possible directions in which to head the boat,

and find the time taken on such courses before the objective is reached.

[Ans. The boat may move either due east or at an angle of $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ due north of east. In the first case, the time is $\frac{6}{\sqrt{2}}$ h and in the second case, the time is $\frac{6}{\sqrt{9+4\sqrt{2}}}$ h.

39. A bus is moving 20 km/h along a straight road and a man, running at 5 km/h along a perpendicular road, sees it when the bus is 150 m away from the junction and he himself is 100 m away from the junction. Show that he can never get nearer to the bus than $\frac{\sqrt{17}}{68}$ km.

Chapter III

Relative Velocity

3.1 Concept of Relative Velocity

The velocity of a moving point P has been defined as the rate of change of its position with respect to some other point, say O. The change in the length and the position of the line OP indicates the movement of the point P as seen from O Similarly, any velocity is measured relative to some point of reference which the observer regards as at rest. Thus if we walk across the deck of a steamer, we should speak of walking at the rate of, say, 3 km/h and we ignore the fact that the steamer is moving through the water at the rate of, say, 30 km/h. When we say that a train is moving at the rate of 75 km/h, we really mean that this is the velocity of the train *relative to the earth*, but we ignore the fact that the earth is also rotating on its axis and carrying the train with it with certain speed. By "actual velocity" or simply "velocity" of a point or an object, we mean the rate of change of its position with respect to another point which is fixed with respect to any other point or object in this universe. But strictly speaking, there is no existence of such a point or object in the earth, because the earth itself is moving around its axis and the sun Thus every motion is *relative* and there is no absolute motion of any point or body. However, for practical problems, by the velocity (or the actual velocity) of a point or a body, we shall always mean the rate of change of its position *with respect to a point fixed on the earth* although this statement, as explained above, is not correct. The velocity of a point or a body will be called *relative* if it changes its position with respect to a moving point or an object on the earth Thus a body will be called *at rest* if it does not change its position with respect to a surrounding object on the earth and it will be called *in motion* when it changes its position with respect to a so-called fixed point or fixed object on the earth, although this statement, as explained above, is also not correct.

3 2 Relative Velocity : Definition

When two points P and Q are both moving in the surface of the earth, then the relative velocity of Q with respect to P is defined as the rate of change of the position of Q as seen from P.

It will be shown that the relative velocity of Q with respect to P is obtained by compounding the velocity of Q with the velocity of P reversed. Similarly, the relative velocity of P with respect to Q is obtained by compounding the velocity of P with the velocity of Q reversed.

Remark . "Relative velocity" is also called "apparent velocity" when the observer is one of the two moving bodies.

3 3 Determination of Relative Velocity from the True Velocities

Let two particles, P and Q be moving with velocities $\vec{V_P}$ and $\vec{V_Q}$ represented by $\vec{AB}$ and $\vec{CD}$ respectively. To find the relative velocity of Q with respect to P, we draw CE equal and parallel to AB and join ED. We note that in unit time, the displacements of P and Q are $\vec{AB}$ and $\vec{CD}$ respectively. Since AB is equal and parallel to CE, the displacement of P can be represented by $\vec{CE}$. Hence, the change of position of Q with respect to P after unit time will be represented by $\vec{ED}$. The rate of change of

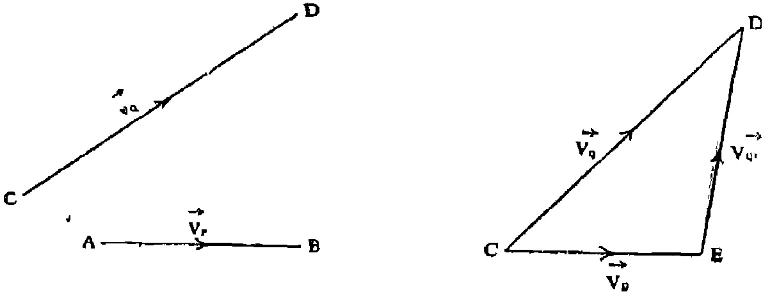


Fig 10

position of Q with respect to P is, therefore, $\vec{ED}$ which, by definition, is the relative velocity of Q with respect to P.

Now, from $\Delta\ CED$,

$\vec{ED}=\vec{CD}+\vec{EC}$

$=\vec{V_Q}-\vec{V_P}$. If we denote the relative velocity of Q with respect to P by $\vec{V_{QP}}$, then $\vec{V_{QP}}=\vec{V_Q}-\vec{V_P}$.

Thus $\vec{V_Q}$ is the resultant of $\vec{V_Q}$ and $\vec{V_P}$ reversed.

Thus we see that *the relative velocity of one moving point with respect to another moving point P is obtained by compounding with the given velocity of Q, a velocity equal and opposite to that of P.*

To find the magnitude and direction of the relative velocity of Q, moving with a velocity of magnitude V_Q with respect to another particle P moving with a velocity of magnitude V_P such that the angle between these two velocities is α, we proceed as follows .

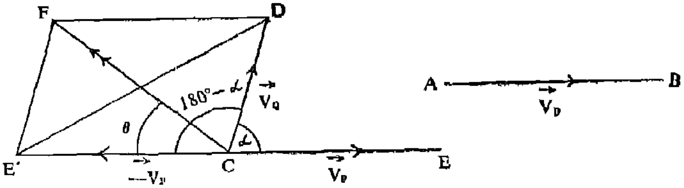


Fig 11

Let $\vec{AB}=\vec{CE}=\vec{V_P}$, $\vec{CD}=\vec{V_Q}$
and $\angle DCE=\alpha$.

We produce EC to E' such that $EC=CE'$. Hence, the magnitude of CE' is V_P and $\angle DCE'=180^\circ-\alpha$. The resultant of the velocities $-\vec{V_P}$ along CE' and $\vec{V_Q}$ along CD will be represented by the diagonal CF of the parallelogram $CE'FD$. Hence, the required relative velocity of Q with respect to P will be represented by CF in magnitude and direction.

$$\therefore \quad V_{QP}=\sqrt{V_P^2+V_Q^2+2V_PV_Q\cos(180^\circ-\alpha)}$$

$$=\sqrt{V_P^2+V_Q^2-2V_PV_Q\cos\alpha}.$$

If $FCE'=0$, then

$$\tan\theta=\frac{V_Q\sin(180^\circ-\alpha)}{V_P+V_Q\cos(180^\circ-\alpha)}=\frac{V_Q\sin\alpha}{V_P-V_Q\cos\alpha}$$

which gives the direction of the relative velocity V_{QP}.

Particular Cases :

(*i*) Let the particles P and Q move parallel to each other in the same direction with velocities u and v $(u>v)$ respectively In this case, $\alpha=0$. Hence, $V_{PQ}=u-v$ in the direction of the velocity of P.

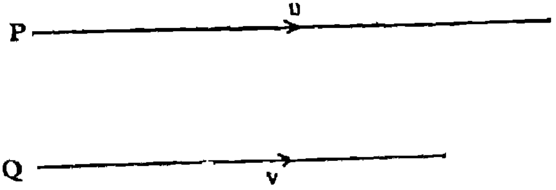


Fig. 12

Thus, to find the relative velocity of P with respect to Q, we simply add with the velocity of P, a velocity equal and opposite to that of Q. If the particles P and Q move with the same velocity in the same direction, then $u=v$ and $\alpha=0$ and so $V_P=0$.

We thus see that if two trains move along two parallel lines with the same velocity in the same direction, then one of the trains will appear to be at rest to a passenger in the other train.

(*ii*) Let the particles P and Q move parallel to each other in opposite directions with velocities u and v respectively. In this case, $\alpha=\pi$. Hence, $V_{PQ}=u+v$ in the direction of the velocity of P. Thus the relative velocity

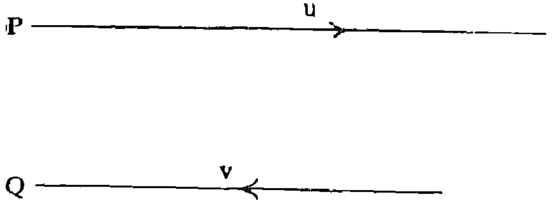


Fig. 13

of P with respect to Q is obtained by adding with the velocity of P, a velocity equal and opposite to that of Q

Thus we see that if two trains move along two parallel lines in the opposite directions, then one of the trains will appear to move faster than its actual velocity to a passenger in the other train.

Remarks (*i*) It may be observed that in finding the relative velocity of one moving point with respect to the other moving point, we actually make the second point at rest (by compounding with the velocity of the first, a velocity equal and opposite to that of the second). We then assume that the first point moves with the resultant of its true velocity and the reversed velocity of the second point as if the second point is at rest. This resultant velocity is actually the required relative velocity of the first point with respect to the second.

(*ii*) When two particles move along two parallel lines or the same line, then the relative velocity of one with respect to the other remains unaltered by the addition of the same velocity to both.

3.4 Determination of True Velocity from the Relative Velocity

Let a point P be moving with velocity $\overrightarrow{V_P}$ represented by $\overrightarrow{AB}$ and let the relative velocity of another point Q with respect to P be $\overrightarrow{V_{QP}}$ represented by $\overrightarrow{ED}$. To find the actual velocity $\overrightarrow{V_Q}$ of Q, we draw EC equal and parallel to AB and complete the parallelogram $ECFD$. Then

$\overrightarrow{EC}=\overrightarrow{V_P}$ and $\overrightarrow{CF}=\overrightarrow{ED}=\overrightarrow{V_{QP}}$

Now, in section 3.3, we have shown that

$$\overrightarrow{V_{QP}}=\overrightarrow{V_Q}-\overrightarrow{V_P}$$

Fig 30

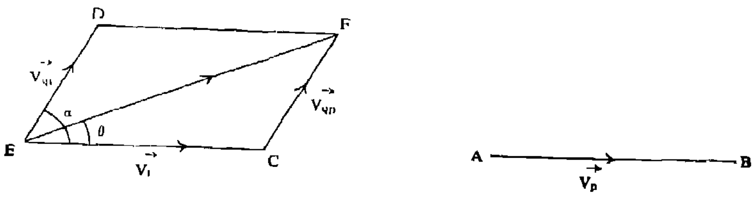


Fig 14

or, $\vec{V_Q}=\vec{V_{QP}}+\vec{V_P}$

$=\vec{EF}$ [from ΔEFC]

Thus the diagonal vector $\vec{EF}$ will represent the actual velocity of Q. Hence, *when the relative velocity of a point Q with respect to P and the true velocity of P are given, then the true velocity of Q is obtained by compounding these two velocities by parallelogram of velocities.*

If $\angle DEC=\alpha$ and $\angle FEC=\theta$, then

$$V_Q^2=V_P^2+V^2_{QP}+2V_PV_{QP}\times\cos\alpha$$

and $$\tan\theta=\frac{V_{QP}\sin\alpha}{V_P+V_{QP}\cos\alpha}.$$

Thus the magnitude and direction of the true velocity of Q are obtained. In solving problems, on relative velocity, we should always remember the following two important results which have been explained in sections 3.3 and 3.4

(a) *Velocity of P relative to Q*

=Resultant of the true velocity of P and the reversed true velocity of Q

(b) *True velocity of Q*

=Resultant of the relative velocity of Q with respect to P and the true velocity of P

3.5 Solved Examples

Ex. 1 One boat is sailing due north at the rate of 12 km/h and another boat is sailing due north-west at the rate of $15\sqrt{2}$ km/hr. Find the magnitude and direction of the velocity of the second boat relative to the first.

Let OA represent the velocity 12 km/h due north and OB represent the velocity $12\sqrt{2}$ km/h due north-west. We reverse the velocity 12 km/h in the direction OC such that $OA=OC$ and complete the parallelogram $OBDC$. Then the diagonal OD represents the velocity of the second boat relative to the first. Now,

$\angle BOC=90°+45°$. Let $\angle DOB=\theta$.

Then $CD^2=OC^2+OB^2+2\times OB\times OC\cos\angle BOC$

$=12^2+(12\sqrt{2})^2+2\times12\times12\sqrt{2}\cos(90°+45°)$

$=144+288-288=144$

$\therefore OD=\sqrt{144}=12.$

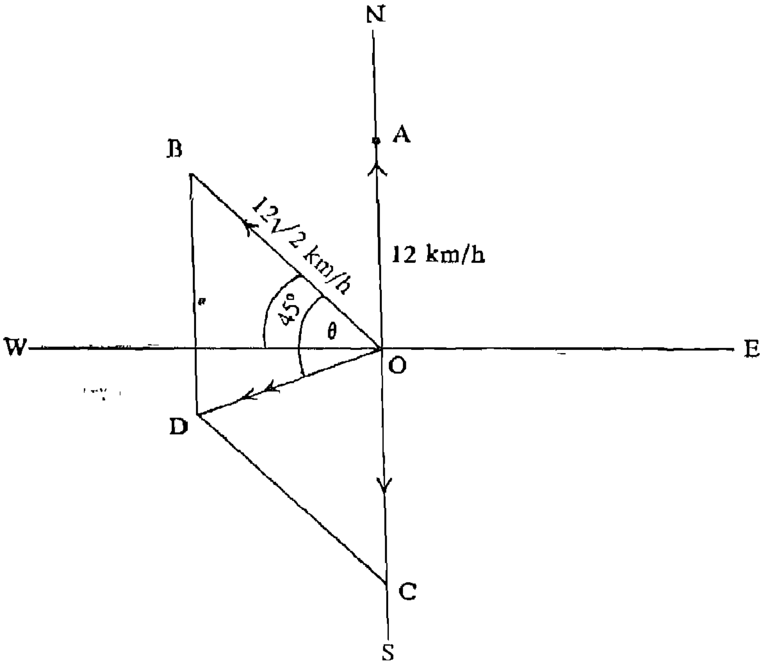


Fig. 15

Also, $\tan\theta = \dfrac{12\sin(90^\circ+45^\circ)}{12\sqrt{2}+12\cos(90^\circ+45^\circ)} = \dfrac{12\times\dfrac{1}{\sqrt{2}}}{12\sqrt{2}-12\times\dfrac{1}{\sqrt{2}}}$

$$= \frac{\dfrac{12}{\sqrt{2}}}{6\sqrt{2}} = \frac{12}{\sqrt{2}} \times \frac{1}{6\sqrt{2}} = 1$$

Hence, $\theta = 45^\circ$.

Thus the required relative velocity is 12 km/h and it makes an angle of 45° with *OB* i.e. the relative velocity is due west.

Ex. 2 An aeroplane is moving due north at 200 km/h relative to a train travelling due east at 60 km/h. Find the velocity of the aeroplane relative to the earth.

Here we shall have to find the true velocity of the aeroplane.

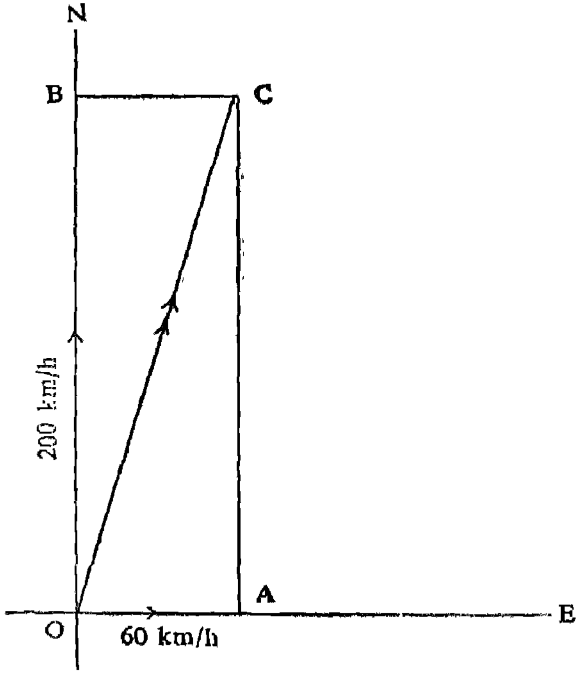


Fig. 16

If OA and OB represent respectively the true velocity of the train and the relative velocity of the aeroplane, then the resultant velocity represented by the diagonal of the rectangle $OACB$ will represent the true velocity of the aeroplane. Hence, the true velocity of the aeroplane

$$=\sqrt{200^2+60^2}\text{ km/h}=20\sqrt{109}\text{ km/h}$$

$$\text{Also}\quad \tan\angle AOC=\frac{200}{60}=\frac{10}{3}$$

$$\therefore\quad \angle AOC=\tan^{-1}\frac{10}{3}.$$

Hence, the direction of the true velocity of the aeroplane will be at an angle of $\tan^{-1}\frac{10}{3}$ due north of east.

Ex. 3. A man is running with a velocity of 6 km/h in a shower of rain which is descending with a velocity of 12 km/h If α be the angle which the apparent direction of the rain makes with the vertical, find tan α.

Let OA represent the reversed velocity of the man in magnitude and OB represent the velocity of the rain.

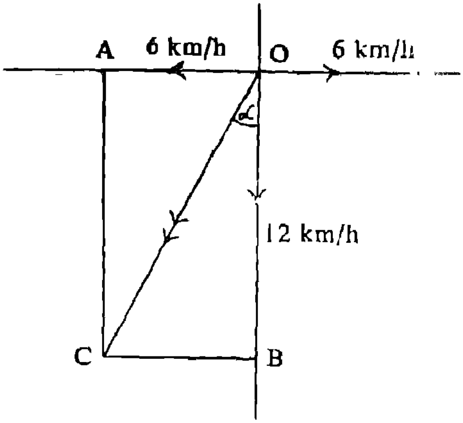


Fig 17

Then $OA = 6$ km/h

and $OB = 12$ km/h.

The diagonal OC of the rectangle $OACB$ will represent the velocity of the rain relative to the man. If $\angle COB = \alpha$, then

$$\tan \alpha = \frac{CB}{OB} = \frac{6}{12} = \frac{1}{2}.$$

Ex. 4 To a man walking at the rate of 3 km/h, rain appears to fall vertically; if he increases his speed to 5 km/h, it appears to fall at an angle of 30° with the vertical Find the actual direction and the velocity of the rain.

Let OA represent the true velocity, 3 km/h, of the man and OC represent the vertical velocity of the rain relative to the man.

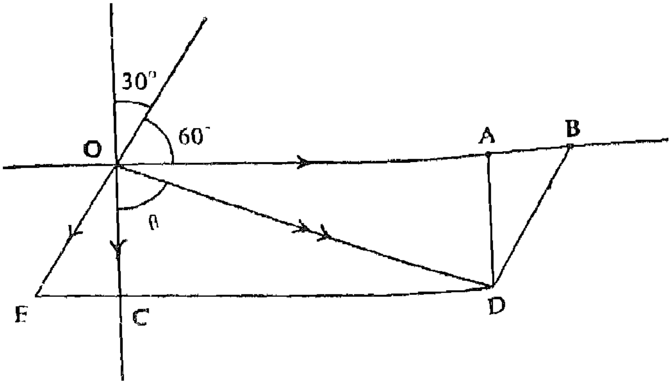


Fig. 18

Since OA is horizontal, hence $\angle AOC=90°$. Hence, the diagonal OD of the rectangle $OADC$ represents the true velocity of the rain When the man increases his speed to 5 km/h, let OB represent this velocity. We join B and D and through D, we draw a line DE equal and parallel to OP to cut OE at E. Then the velocity of the rain relative to the man will now be represented by OE where $OE \parallel BD$ and OE makes an angle of 30° with the vertical OC, so that the diagonal OD of the parallelogram $OEDB$ represents the true velocity of the rain, as before.

Let $\angle DOC=\theta$ We shall find OD and θ. Let $OD=u$ so that u km/h is the actual velocity of the rain. Let v_1 km/h and v_2 km/h represent the relative velocities of the rain in the first and the second cases respectively so that $OC=v_1$ and $OE=v_2$ In the first case, u km/h is the resultant of 3 km/h along OA and v_1 km/h along OC. Hence, the resolved part of the resultant u km/h in any direction will be equal to the algebraic sum of the resolved parts of the component velocities 3 km/h and v_1 km/h.

Resolving perpendicular to the direction of v_1, we get

$$u \sin \theta=3 \qquad \ldots(1)$$

In the second case, we resolve the velocities u km/h, v_2 km/h and 5 km/h perpendicular to the direction of v_2 km/h. Thus we get

$$u \sin (\theta+30°)=5 \sin 60°. \qquad \ldots(2)$$

From (1) and (2), we shall solve for u and θ.

Dividing (2) by (3), we get

$$\frac{\sin (\theta+30°)}{\sin \theta}=\frac{5 \sin 60°}{3}$$

$$\Rightarrow \frac{\sin\theta\cos 30^\circ+\cos\theta\sin 30^\circ}{\sin\theta}=\frac{5}{3}\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{\sqrt{3}}{2}+\frac{1}{2}\cot\theta=\frac{5\sqrt{3}}{6}$$

$$\Rightarrow \frac{1}{2}\cot\theta=\frac{2\sqrt{3}}{6}=\frac{\sqrt{3}}{3}\Rightarrow\cot\theta=\frac{2}{\sqrt{3}}\Rightarrow\quad\tan\theta=\frac{\sqrt{3}}{2}$$

$$\therefore\quad \sec^2\theta=1+\tan^2\theta=1+\frac{3}{4}=\frac{7}{4}$$

$$\Rightarrow\quad \cos^2\theta=\frac{4}{7}\Rightarrow\sin^2\theta=1-\frac{4}{7}=\frac{3}{7}$$

$$\Rightarrow\quad \sin\theta=\sqrt{\frac{3}{7}}.\qquad[\therefore\sin\theta>0]$$

Hence, from (1)

$$u\times\sqrt{\frac{3}{7}}=3$$

$$\Rightarrow u=\frac{3\sqrt{7}}{\sqrt{3}}=\sqrt{21}\simeq 4.58.$$

Hence, the required true velocity is 4.58 km/h making an angle $\tan^{-1}\frac{\sqrt{3}}{2}$ with the downward vertical.

Ex. 5 A bird flies at the rate of 22 km/h in still air. The wind is blowing from south-west at 9 km/h. In what direction must the bird try to fly in order that it may actually go north-west, and what will be its velocity in the north-west direction ?

The velocity of the bird in still air is 22 km/h This means that the velocity of the bird relative to air is 22 km/h i.e. $V_{BA}=9$ km/h making an angle of 45° with east. It is given that the direction of the true velocity of the bird, V_B, is due north-west. To determine the magnitude of V_B and the direction of V_{BA}, let OC represent V_B and OA represent V_A reversed so that the diagonal OB of the rectangle $OABC$ represents $V_{BA}=22$ km/h. Hence, $OA=9$ km/h, $OB=22$ km/h and $\angle AOC=90^\circ$.

Hence, from $\triangle\ AOB$, $AB=\sqrt{22^2-9^2}=\sqrt{403}$

$$\simeq 20.1$$

$$\therefore\quad OC=AB\simeq 20.1.$$

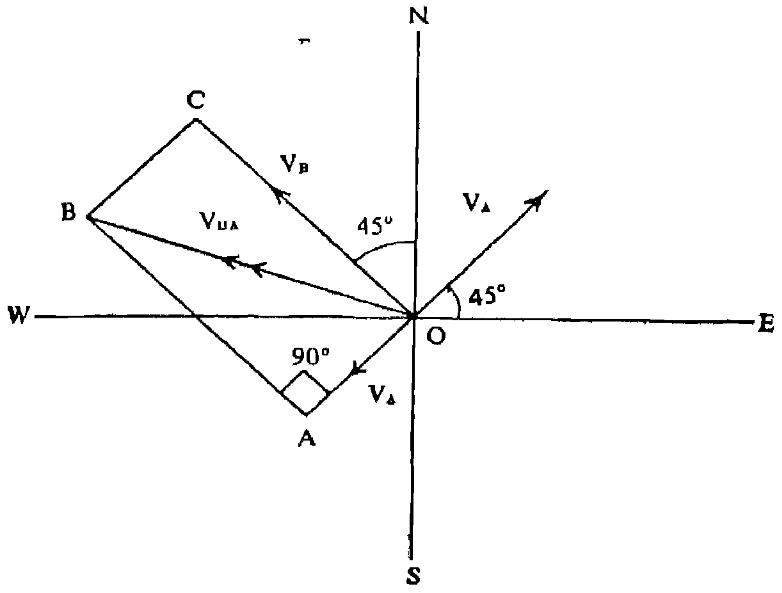


Fig. 19

If $\angle AOB = \theta$, then $\cos\theta = \frac{9}{22}$

Hence, the magnitude of the true velocity of the bird is 20.1 km/h approximately and the direction of the velocity of the bird relative to air makes an angle of $\cos^{-1}\left(\frac{9}{22}\right)$ with the south-west direction.

Ex. 6. To an observer on a ship travelling due west at 16 km/h, another ship 1 km due south appear to be travelling north-east at 12 km/h. Find the magnitude and direction of the velocity of the second ship, and the distance apart of the two ships when nearest to each other.

Here, the true velocity of the first ship is given to be 16 km/h due west and the velocity of the second ship relative to the first is given to be 12 km/h due north-east. We have to find the true velocity of the second ship and the least distance between the two ships.

Let AB represent 12 km/h making an angle of 45° with OA and AC represent 16 km/h. Then the diagonal AD of the parallelogram

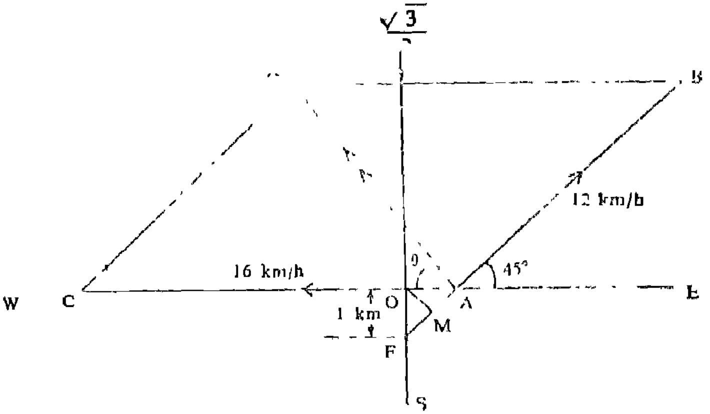


Fig- 20

ACDB represents the true velocity of the second ship.

Now, $AD=\sqrt{12^2+16^2+2\times 12\times 16 \cos(180^\circ-45^\circ)}$

$$=\sqrt{400-192\sqrt{2}}\simeq\sqrt{400-271.4818}$$
$$=\sqrt{128.512}$$
$$\simeq 11.34$$

Hence, the true velocity of the second ship is 11.34 km/h approximately.

Let $\angle DAC=\theta$.

$$\therefore \quad \tan\theta=\frac{12\sin 45^\circ}{16-12\cos 45^\circ}=\frac{6\sqrt{2}}{16-6\sqrt{2}}$$
$$\simeq 1.129$$

$\therefore \quad \theta\simeq\tan^{-1} 1.129.$

Hence, the direction of the true velocity of the second ship will make an angle of $\tan^{-1}(1.129)$ approximately with the west

To find the least distance between the two ships, we can assume that the first ship is at rest and the second ship is sailing along *FA* due north-east relative to the first ship, where it is given that *OF*=1 km.

We draw *OM* perpendicular to *FA*, the direction of the relative motion of the second ship. Then *OM* will be the required minimum distance between the two ships. Now, from $\triangle OFM$, we have

$$OM = OF \sin \angle OFM = \sin 45^\circ = \frac{1}{\sqrt{2}} \simeq 0.71.$$

Hence, the least distance between the two ships is 0.71 km nearly.

Ex. 7 A battleship leaves a port P and sails northwards. A submarine simultaneously starts from a point Q, 50 km east of P with the intention of having the ship within 25 km range of itself. If the ratio of the velocities of the battleship and the submarine be $2 : \sqrt{6}$, find the extreme directions within which it must direct its motion.

Let the velocity of the submarine be $\sqrt{6}\,t$ km/h and that of the battleship be $2t$ km/h. The distance of the battleship at any time from

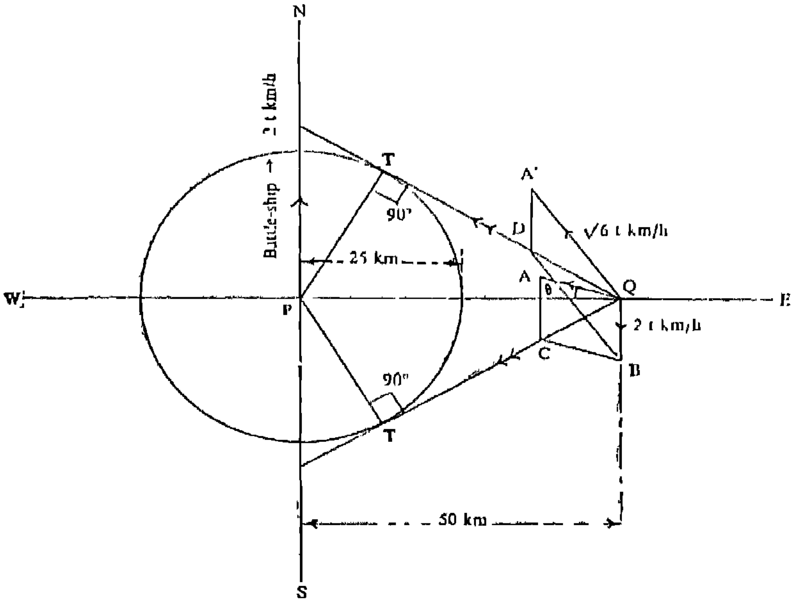


Fig: 21

the submarine may be obtained by reducing the former to rest and making the latter move with the relative velocity. Let $QA = QA' = \sqrt{6}\,t$ km/h represent the velocity of the submarine in a direction, say, θ° north of west and $QB = 2t$ km/h represent the reversed velocity of the bettleship due south. The resultant of QA and QB will give the relative velocity of the submarine with respect to the battleship. It is given that $PQ=50$ km. We draw a circle of radius 25 km with centre P and let QT and QT' be the tangents to it from Q. In order that the submarine may get the battleship within 26 km of itself, the direction of its relative velocity must be within the angle TQT' Hence, the extreme directions of QA (or QA') and QB may be QT in one case and QT' in the other case.

$$\text{Now,}\quad \sin \angle PQT=\frac{PT}{PQ}=\frac{25}{50}=\frac{1}{2}$$

$$\therefore\quad \angle PQT=30^\circ \qquad \therefore\quad \angle PQT'=30^\circ$$

$$\text{Thus } \angle BQT=90^\circ+30^\circ=120^\circ$$

and $$\angle BQT'=90^\circ-30^\circ=60^\circ$$

Now, since $\angle BQA=90^\circ+\theta$ and the resultant of QB and QA is QC which makes an angle of 60° with QB, we have

$$\tan 60^\circ=\frac{\sqrt{6}\,t \sin(90^\circ+\theta)}{2t+\sqrt{6}\,t\cos(90^\circ+\theta)}$$

$$\Rightarrow\quad \sqrt{3}=\frac{\sqrt{6}\cos\theta}{2-\sqrt{6}\sin\theta}$$

$$\Rightarrow\quad 2\sqrt{3}-3\sqrt{2}\sin\theta=\sqrt{6}\cos\theta$$

$$\Rightarrow\quad \sqrt{6}\cos\theta+3\sqrt{2}\sin\theta=2\sqrt{3}$$

$$\Rightarrow\quad \frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta=\frac{1}{\sqrt{2}} \quad [\text{Dividing by } 2\sqrt{6}\,]$$

$$\Rightarrow\quad \sin\left(\theta+\frac{\pi}{6}\right)=\sin\frac{\pi}{4}$$

$$\Rightarrow\quad \theta=\frac{\pi}{4}-\frac{\pi}{6}$$

$$=\frac{3\pi-2\pi}{12}$$

$$=\frac{\pi}{2}$$

$$=15^\circ.$$

Again, since the resultant of QB and QA' is QD which makes an angle 120° with QB, we have

$$\tan 120^\circ=\frac{\sqrt{6}\,t\sin(90^\circ+\theta)}{2t+\sqrt{6}\,t\cos(90^\circ+\theta)}$$

$$\Rightarrow \quad -\sqrt{3} = \frac{\sqrt{6}\cos\theta}{2-\sqrt{6}\sin\theta}$$

$$\Rightarrow \quad -2\sqrt{3}+3\sqrt{2}\sin\theta = \sqrt{6}\cos\theta$$

$$\Rightarrow \quad \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\left(\theta - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12} = 75^\circ.$$

Hence, the required extreme directions of the submarine are 15° north of west or 75° north of west.

Ex. 8. If d be the distance at any time between two points moving uniformly in one plane, V, the relative velocity of one with respect to the other and u, v, the resolved parts of V along and perpendicular to the direction of the line joining the two points, show that their distance when they are nearest to each other is $\frac{vd}{V}$, and the time of arriving at this nearest distance is $\frac{ud}{V^2}$.

Let AC and BF represent the true velocities of the points A and B respectively. We draw AE equal and parallel to BF but in the opposite directions. Then the diagonal AD of the parallelogram $ACDE$ will represent the velocity V of A relative to B. We draw $DN \perp AB$ and $AG \perp AB$ so that $ANDG$ is a rectangle. Hence, u and v, the components of V, can be represented by AN and AG respectively. We draw $BM \perp AD$ produced. Let $\angle BAM = \theta$.

It is given that $AB = d$.

Since AD represents the velocity of A relative to B, the point B can be thought to be at rest and the point A moving along AM with velocity V. Hence, the minimum distance between A and B is given by BM.

Now, from $\triangle ABM$,

$$BM = AB\sin\theta = d\sin\theta.$$

But from $\triangle ADN$,

$$\sin\theta = \frac{DN}{AD} = \frac{v}{V}$$

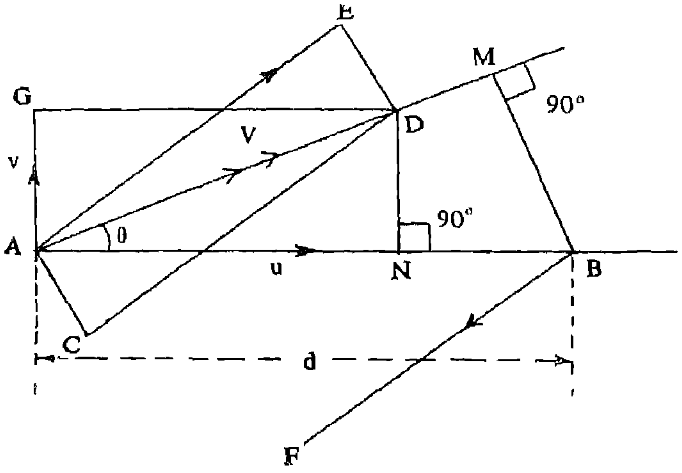


Fig 22

Hence, $BM=\frac{vd}{V}$ i e. the minimum distance is $\frac{vd}{V}$. Also, the time of describing the distance AM by the point A will be

$$\frac{AM}{V}=\frac{d\cos\theta}{V} \qquad \left[\text{From } \triangle\, ABM\right]$$

$$=\frac{d}{V}\times\frac{u}{V} \qquad \left[\text{From } \triangle\, ADN\right]$$

$$=\frac{ud}{V^2}.$$

Hence the results.

Ex. 9. A line of men are running along a road at 8 km/h behind one another at equal intervals of 20 m. A line of cyclists are riding in the same direction at 15 km/h at equal intervals of 30 m. At what speed must an observer travel along the road in the opposite direction so that whenever he meets a runner, he also meets a cyclist.

Let the cyclists and runners be moving along parallel lines A_1B_1 and A_2B_2.

Let C', C'' be the two positions of two cyclists and R', R'' be those of two runners respectively after some time so that $C'\,C''=30$ m and $R'\,R''=20$ m.

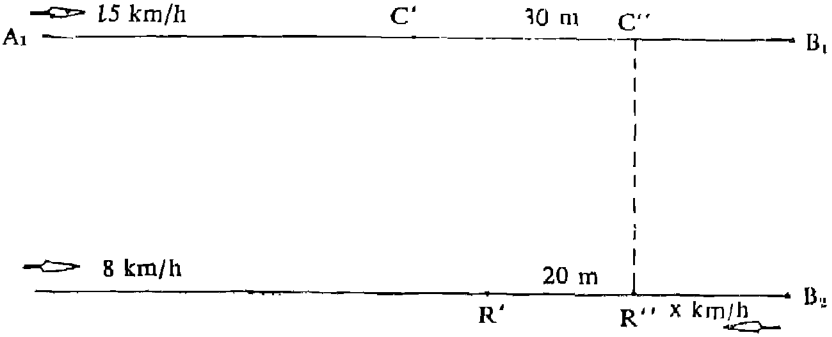


Fig 23

Let x km/h be the velocity of the observer in the opposite direction. We now apply a reversed velocity of x km/h to each velocity of each cyclist and each runner so as to bring the observer to rest. Each cyclist and each runner will be moving with velocities $(15+x)$ km/h and $(8+x)$ km/h respectively relative to the observer. If at any time the observer meets the runner and the cyclist simultaneously when they are at, say, C'' and R'' respectively, then the observer will meet the next cyclist and the next runner after times $\frac{30}{1000\,(15+x)}$ h and $\frac{20}{1000\,(8+x)}$ h respectively.

If the observer meets the second cyclist and the second runner at the same time, then

$$\frac{30}{1000\,(15+x)}=\frac{20}{1000\,(8+x)}$$

$$\Rightarrow\ 24+3x=30+2x$$

$$\Rightarrow\ x=6.$$

Hence, the required velocity of the observer will be 6 km/h in a direction opposite to the motion of the cyclists and the runners.

Ex. 10. Three straight lines are equally inclined to one another in a plane and three particles P, Q, R move along them with velocities u, v, w respectively. Show that the velocity of P relative to the centroid of the particles is

$$\frac{1}{3}\,(4u^2+v^2+w^2+2uv+2uw-vw)^{\frac{1}{2}}\ .$$

[Note that this problem can be solved easily be vector method which is discussed below.]

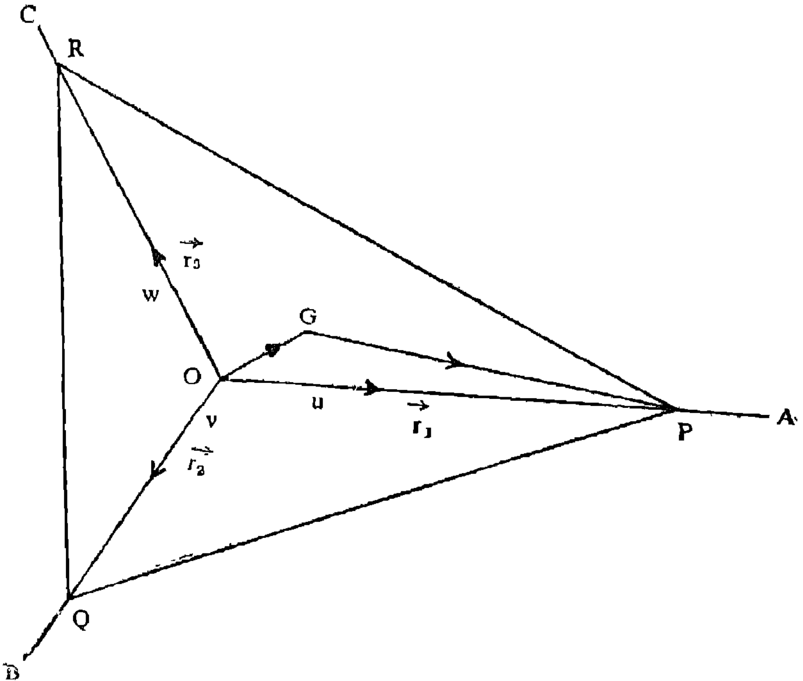


Fig. 24

Let OA, OB and OC be three lines which are equally inclined to one another so that $\angle AOC=\angle BOC=\angle BOA=120°$.

Let $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ be the position vectors of P, Q and R on OA, OB and OC respectively at any time t with respect to O. Let G be the centroid of Δ PQR so that $\vec{OG}=\frac{1}{3}(\vec{r}_1+\vec{r}_2+\vec{r}_3)$. We join PG. Then the position vector of P relative to G

$$=\vec{GP}=\vec{OP}-\vec{OG}=\vec{r}_1-\frac{1}{3}(\vec{r}_1+\vec{r}_2+\vec{r}_3)$$

$$=\frac{1}{3}(2\vec{r}_1-\vec{r}_2-\vec{r}_3)$$

If $\vec{GP}=\vec{r}$, i.e. the position vector of P relative to G, then

$$\vec{r}=\frac{1}{3}\,(2\,\vec{r_1}-\vec{r_2}-\vec{r_3})$$

$$\therefore\ \frac{d\,\vec{r}}{dt}=\frac{1}{3}\left[2\,\frac{d\,\vec{r_1}}{dt}-\frac{d\,\vec{r_2}}{dt}-\frac{d\,\vec{r_3}}{dt}\right]$$

$$=\frac{1}{3}\,(2\vec{u}-\vec{v}-\vec{w})$$

$$\Rightarrow\ \vec{V_{PG}}=\frac{1}{3}\,(2\,\vec{u}-\vec{v}-\vec{w})$$

$$\Rightarrow\ V^2_{PQ}=\frac{1}{9}\,(4u^2+v^2+w^2-4\,\vec{u}\cdot\vec{v}-4\,\vec{u}\cdot\vec{w}+2\,\vec{v}\cdot\vec{w})$$

$$=\frac{1}{9}\,(4u^2+v^2+w^2-4uv\cos 120^\circ-4uw\cos 120^\circ+2vw\cos 120^\circ)$$

$$=\frac{1}{9}\,(4u^2+v^2+w^2+2uv+2uw-vw)$$

$$\Rightarrow\ V_{PG}=\frac{1}{3}\,(4u^2+v^2+w^2+2uv+2uw-vw)^{\frac{1}{2}}\ .$$

Hence the result.

EXERCISE 3

1. A school boy holding an umbrella runs with a velocity equal to that of the rain falling vertically, in consequence of which the rain strikes him in the face. At what angle should he hold the umbrella in order to protect him best ?

   [Ans. 45° with the vertical]

2. A steamer is going due north with velocity v. The smoke from the chimney points 45° south of east. If the wind be coming from west, find the magnitude of its velocity. [Assume that the smoke from the chimney loses the velocity of the steamer as

soon as it leaves the chimney and moves with the velocity of the wind.] [Ans. v]

3. Two points A and B move with speeds u and $2u$ respectively in two straight lines inclined at an angle α. Show that the relative velocity of B with respect to A is

$$u\sqrt{5-4\cos\alpha}\,.$$

4. A man wishes to cross a river to an exactly opposite point on the other bank. If he can swim with twice the velocity of the current, find at what inclination to the current he must swim. [Ans. 120°]

5. A person travelling towards the north-east finds that the wind appears to blow from the north, but when be doubles his speed, it seems to come from a direction making an angle $\tan^{-1}\frac{1}{2}$ east of north. Find the true direction of the wind. [Ans. Towards the east]

6. Three points P, Q and R move with the same velocity v along the sides BC, CA and AB respectively of an equilateral triangle. Find the velocity of any one relative to any other in magnitude and direction [Ans $v\sqrt{3}$ parallel to an attitude]

7. Given the relative velocity of A with respect to B and also the relative of B with respect to C Show how you will proceed to find the relative velocity of C with respect to A.
[Ans. By combining the two given relative velocities and reversing the direction of the resultant]

8. A steamer is travelling due east at the rate of u km/h A second steamer is travelling at $2u$ km/h in a direction θ° north of east and appears to a passenger on the first steamer to be travelling north-east. Show that

$$\theta=\frac{1}{2}\sin^{-1}\frac{3}{4}$$

9. When a man runs at 6 km/h due south, the wind appears to him to blow from the east. When the man doubles his speed, the wind appears to blow from the south-east. Find the velocity of the wind [Ans. $6\sqrt{2}$ km/h from north-east]

10. A man falling vertically by parachute in a steady downpour of rain observes that when his speed is v_1, the rain appears to make an angle α with the vertical. When his speed is v_2, the

rain appears to make an angle β with the vertical. Show that the rain actually falls at an angle θ with the vertical given by $(v_2-v_1)\cot\theta=v_2\cot\alpha-v_1\cot\beta$.

11. The velocity of a body B relative to a body A has components 3 m/s towards A and of 4 m/s in the direction perpendicular to AB. If $AB=100$ m, show that the least distance between the bodies will be 80 m. When will A and B be at this distance apart? [Ans. After 12 s]

12. Two roads cross at right angles. A motor car is 2 km from the intersection and is travelling towards it on the first road at 21 km/h; at the same instant, another car is 3 km from the intersection and is travelling towards it on the second road at 28 km/h. If these velocities are maintained, show that the distance between the two cars will never be less than $\frac{1}{5}$ km.

13. A river is flowing from west to east at 2 km/h and a boat is rowed with a velocity of 4 km/h due south relative to the current. A hackney carriage runs on a road parallel to the river towards the west at the rate of 6 km/h. Find the apparent velocity of the boat as seen by an observer on the carriage.

$\left[\text{Ans. } 4\sqrt{5} \text{ km/h at } \tan^{-1}\frac{1}{2} \text{ south of east.}\right]$

14. At a particular instant two aeroplanes are at a distance of 1000 km, one due east of the other. The first one is moving westwards at the rate of 400 km/h and the second with a velocity of 300 km/h towards the south. Find the time when they are nearest to each other, and the least distance between them. [Ans. 1 h 36 min; 600 km]

15. At 1 P.M. an animal A is $10\sqrt{2}$ km due west of another animal B The first runs north-east at 12 km/h and the second north-west at 16 km/h. How fast do they approach each other and when are they nearest?

[Ans. At 20 km/h; at 1-42 P.M.]

16. To a cyclist travelling at 1 km/h due east, the wind appears to come from the north-east. But when he travels north-east at the same speed, it appears to come from the north. Find the true direction and the velocity of the wind.

$\left[\text{Ans. } 22\frac{1}{2}^{\circ} \text{ south of east ; } 8\sqrt{2-\sqrt{2}} \text{ km/h}\right]$

17 A cyclist is riding due west at a speed of 12 km/h and the wind is at the time blowing from the south-east with a speed of $5\frac{1}{2}$ km/h. If the cyclist carries a small flag, in which direction will this flag fly ? At what speed would the cyclist require to ride, if the flag is to fly due north ?

[Ans. $\tan^{-1}\frac{264\sqrt{2}+121}{1031}$ due north of east,

$\frac{11\sqrt{2}}{4}$ km/h]

18. A motor car moving at the rate of 60 km/h towards east is struck by a stone moving horizontally from the north with a velocity of 30 m/s. Find the magnitude and direction of the velocity with which the stone appears to meet the car

[Ans $\frac{10\sqrt{106}}{3}$ m/s in a direction making an angle $\tan^{-1}\frac{9}{5}$ south of west]

19. A man can row at 4 km/h in still water (i.e. the man can row at 4 km/h relative to water). If he steers straight across a stream which is running at 3 km/h, find his actual velocity stating the angle which his direction makes with that of the stream. [Ans. 5 km/h ; $\tan^{-1}\frac{4}{3}$]

20. From a train running slowly at 8 km/h, a man jumps at 10 km/h in a direction inclined at 60° to the direction of motion of the train. Find the actual velocity, giving the angle it makes with the direction of the train.

[Ans. $2\sqrt{61}$ km/h; $\tan^{-1}\frac{5\sqrt{3}}{13}$]

21. Two roads cross at right angles at P. A man, A, walking along one of them at 3 km/h towards P sees another man B, walking on the other road at 4 km/h at P when A is $\frac{5}{88}$ km away from P. Find the velocity of A relative to B and show that they will be nearest together when A has walked $\frac{9}{440}$ km.

[Ans 5 km/h, $\tan^{-1}\frac{4}{3}$ with A's direction]

22. A submarine leaves a port X and travels at 10 km/h due south-west. At the same instant, a destroyer leaves another port Y which is 20 km south of X. If the destroyer can travel at 25 km/h, in which direction should it move to ram the submarine? Find also the time taken.

$$\left[\text{Ans. } \sin^{-1}\frac{\sqrt{2}}{5} \text{ due west of south ; } \frac{4(\sqrt{23}-\sqrt{2})}{21} \text{ hours}\right]$$

23. A cyclist is riding due west at a speed of 12 km/h and the wind is at the time blowing from the south-east with a speed of $5\frac{1}{2}$ km/h If the cyclist carries a small flag, in which direction will this flag fly? At what speed would the cyclist require to ride, if the flag is to fly due north?

$$\left[\text{Ans. } \tan^{-1}\frac{264\sqrt{2}+121}{1031} \text{ due north of east ; } \frac{11\sqrt{2}}{4} \text{ km/h}\right]$$

24. A man wishes to swim across a river in the least time. If the velocity of flow of the river and the velocity with which the man can swim in still water be equal, find in what direction he should start to swim and the path which he would describe.

[Ans. Perpendicularly. The path is a line inclined at 45° to the direction of flow]

25. To an observer on a train moving at 30 km/h due north, wind appears to blow from 15° east of north, and from a motor car running at $15(\sqrt{3}-1)$ km/h due east, it appears to come from 15° north of east. Find the true direction of the wind.

[Ans. From 30° north of east]

26. Two roads cross at an angle of 60°. Two persons, one on each road walking at the same speed, are approaching the crossing (acute angle), their simultaneous distances being 100 m and 200 m respectively. Find their distances from the crossing at the instant when they are nearest to one another.

[Ans. 50 m each]

27. A battleship which can steam at u km/h sights an enemy cruiser at a distance a km due east of her. If the cruiser steams due north at v km/h ($v>u$), find in which direction the battleship must steer in order to get as close to the cruiser as possible,

and show that when they are as close as possible, the battleship will have travelled $\frac{au}{v}$ km relative to the cruiser.

[Ans. $\cos^{-1}\frac{u}{v}$ east of north]

28. A cyclist riding at a speed V, overtakes a pedestrian who can move at a speed not greater than V, the two travelling along parallel tracks at a distance d apart. Show that if the cyclist rings his bell when at a distance less than $\frac{V}{v}d$, he may safely maintain his speed and keep to his course regardless of the behaviour of the pedestrian.

29. A bomber moves due east at 100 km/h over a town at a certain time. Six minutes later, a pursuit plane starts from a station Y which is 40 km due south of X and flies north-east. If both maintain their course, find the magnitude of the velocity with which the pursuit must fly in order to overtake the bomber.

[Ans. $\frac{400\sqrt{2}}{3}$ km/h]

30. A man pulls a boat at 4 km/h in a direction at right angles to the bank of a river, and is carried by the current which flows at 3 km/h. Find the velocity of the boat with respect to the bank of the river. If the river is $\frac{1}{2}$ km broad, find the time taken to cross, and find how far down the stream the boat is carried. What distance does the boat actually travel ? How long would it take for the boat to cross the river if there were no current ?

[Ans. Velocity of the boat = 5km/h in a direction making an angle $\tan^{-1}\frac{4}{3}$ with the direction of the current Time for crossing the river = $7\frac{1}{2}$ minutes. The boat will be carried through a distance of $\frac{3}{8}$ km down the stream. The distance actually travelled = $\frac{5}{8}$ km. The time taken by the boat to cross the river in absence of the current = $7\frac{1}{2}$ minutes]

31. A railway train is moving at the rate of $28\,t$ km/h when a pistol shot strikes it in a direction making an angle $\sin^{-1}\frac{3}{5}$ with the train. The shot enters the compartment at the corner furthest from the engine and passes out at the diagonally opposite corner. If the compartment is $8k$ m long and $6k$ m wide along the track, show that the velocity of the shot is $80\,t$ km/hr and traverses the compartment in $\left(\frac{3}{5}\cdot\frac{k}{t}\right)$ seconds.

32. Two boats move with constant speed v relative to the water, and both cross a straight river of constant width a, which flows with constant speed V. One crosses the river by the shortest path and the other in the shortest time. Prove that the difference between the times taken to cross the river is

$$\frac{a}{u}\left(\frac{u}{\sqrt{u^2-V^2}}-1\right) \text{or } \frac{a}{u}\left(\frac{V}{\sqrt{V^2-u^2}}\quad 1\right)$$

according as u or V is greater.

33. Two cars A and B on different roads at right angles are approaching the junction at such speeds that they would collide. What can you infer about the velocity of A relative to B ?

    To avoid a collision, A drops his speed by 10 km/h for 3 s, but then continues at his original speed. How far is A from the junction when B reaches it ? If A's original speed is twice B's, how far will the cars be apart when they are nearest to each other. [Ans. Parallel to AB ; 44 m ; 19 7 m]

34. A ball travelling horizontally at 60 m/s is struck by a bat and made to travel in the direction at right angles to its original path with a speed of 83 m/s. Find the velocity which the bat would have communicated to the ball had it been at rest giving the angle this velocity makes with the original direction of the ball. $\left[\text{Ans. } 100 \text{ m/s} ; \tan^{-1}\frac{4}{3}\right]$

35 A ferryman rows a boat at 5 km/h in a north-east direction relative to the water of a river flowing south at 2 km/h. What is the velocity of the boat relative to the bank of the river ?

$$\left[\text{Ans. } \sqrt{29-10\sqrt{2}} \text{ km/h in a direction } \left(\frac{\pi}{4}+\tan^{-1}\frac{5\sqrt{2}+2}{23}\right) \text{ east of north}\right]$$

Chapter IV

Motion in a Straight Line with Acceleration

4.1. Expressions for Velocity and Acceleration in Terms of Displacement and Time

Let a particle be moving along a straight line and let x, v and a be its displacement, velocity and acceleration respectively at any time t. As remarked in section 1.2 of chapter I, x, v and a are, in fact, the "magnitudes" of the displacement vector $\vec{x}$, the velocity vector $\vec{v}$ and the acceleration vector $\vec{a}$. In order to avoid vector calculus, we sha' use the term "velocity", "acceleration" and other physical quantities in the sense of their magnitudes only.

As discussed in section 1.2 in chapter 1,

$$v=\frac{dx}{dt} \tag{1}$$

$$a=\frac{dv}{dt}. \tag{2}$$

Now, putting $v=\frac{dx}{dt}$ in (2), we get

$$a=\frac{d}{dt}\left(\frac{dx}{dt}\right)=\frac{d^2x}{dt^2} \tag{3}$$

Again, from (2), we have

$$a=\frac{dv}{dx}\times\frac{dx}{dt}=v\ \frac{dv}{dx}. \tag{4}$$

Hence, the velocity is given by (1) and the acceleration is given by any one of the equations (2), (3) and (4). It follows that both velocity and acceleration are functions of time t which, in turn, is a function of displacement x. If the value of a is negative, we say that the particle moves with retardation i.e. its velocity gradually decreases. Hence, the negative acceleration decreases. Thus the negative acceleration is called retardation.

4.2. Motion in a Straight Line with Constant Acceleration

A particle moves along a straight line from a fixed point on it with initial velocity u and constant acceleration a. To find

(*i*) *the velocity v at any time t* and

(*ii*) *the displacement s at any time t* and

(*iii*) *the velocity when the displacement is s.*

(i) Using the equation

$$\frac{dv}{dt}=a,$$

we get, by integration,

$$v=\int a\,dt=at+c, \quad \ldots(1)$$

where c is a constant of integration.

Initially, $v=u$ when $t=0$.

Hence, $u=c$.

Hence, from (1), we get

$$v=u+at \quad \ldots(2)$$

The equation (2) gives us v in terms of u, a and t.

(ii) To express the displacement s in terms of t, we start from the equation

$$\frac{d^2s}{dt^2}=a.$$

Integrating both sides with respect to t, we get

$$\frac{ds}{dt}=\int a\,dt=at+c_1, \quad (3)$$

where c_1 is a constant of integration.

Initially, $\frac{ds}{dt}$ i.e. velocity $=u$ when $t=0$.

Hence, from (3), we get

$$u=c_1$$

Substituting the value of c_1 in (3), we get

$$\frac{ds}{dt}=at+u \qquad \ldots(4)$$

Integrating (4) again with respect to t, we have

$$s=\int(at+u)\,dt$$

$$=\frac{at^2}{2}+ut+c_2, \qquad \ldots(5)$$

where c_2 is a constant of integration.

Initially, $s=0$ when $t=0$.

Hence, from (5), $c_2=0$.

Thus we get

$$s=ut+\frac{1}{2}at^2 \qquad \ldots(6)$$

The equation (6) gives the displacement s in terms of u, t and a.

(iii) To find the velocity v in terms of the displacement s, we can eliminate t from (2) and (6). Alternatively, we can directly deduce the relation starting from the equation

$$\frac{vdv}{ds}=a \Rightarrow vdv=ads.$$

Integrating both sides of this equation with respect to the variables involved, we get

$$\int vdv=\int ads$$

$$\Rightarrow\frac{v^2}{2}=as+c_1, \qquad \ldots(7)$$

where c_1 is a constant of integration.

Now, initially, $v=u$ when $s=0$.

Hence, from (7), we get

$$\frac{u^2}{2}=c_1.$$

Substituting this value of c_1 in (7), we get

$$\frac{v^2}{2}=as+\frac{u^2}{2}$$

$$\therefore \quad v^2=u^2+2\,as \tag{8}$$

The equations (2), (6) and (8) are called the equations of motion of a particle in a straight line with a constant acceleration.

In particular, if the particle starts from rest, then $u=0$ so that the equations (2), (6) and (8) reduce to

$$v=at,\quad s=\frac{1}{2}at^2,\quad v^2=2\,as$$

Remark : The above formulae are applicable only when the acceleration is uniform or constant In the case of variable acceleration, we must have recourse to calculus. This will be explained with solved examples

4.3. Distance Described in a Particular Time

To deduce the distance described by a particle moving with an initial velocity u and a uniform acceleration a, at a particular time, say, in tth second, we subtract the distance described in $(t-1)$ s from that described in t s. If S_t denote this distance in tth second, then

$$S_t=\left(ut+\frac{1}{2}at^2\right)-\left[u\,(t-1)+\frac{1}{2}a(t-1)^2\right]$$

$$=u+\frac{1}{2}a(2t-1).$$

In particular, if the initial velocity $u=0$, then

$$S_t=\frac{1}{2}a\,(2t-1).$$

4.4. Average Veloctiy

The average velocity V of a particle moving along a line with a constant acceleration is given by

$$V=\frac{\text{total distance}}{\text{total time required}}$$

$$=\frac{S}{t}=\frac{ut+\frac{1}{2}at^2}{t}$$

$$=u+\frac{1}{2}at \qquad (1)$$

Comparing this equation with the equation

$v=u+at$, we see that

V is the velocity at time $\frac{t}{2}$ i.e at the middle of the interval of time.

Hence, *the average velocity of a particle moving along a straight line with a constant acceleration during time t is the velocity at time $\frac{t}{2}$.*

Again, the equation (1) can be re-written as

$$V=\frac{2u+at}{2}=\frac{u+(u+at)}{2}$$

$$=\frac{u+v}{2}, \qquad (2)$$

where v is the velocity of the particle at time t. Hence, *the average velocity is also defined as the mean of the initial and the final velocities at time t.*

4.5. Working Rules for Solving Problems

Most of the problems of the motion of a particle on a straight line with a uniform acceleration will require the equations of motion (2), (6) and (8) of section 4.2 for their solution. It can be observed that these equations of motion involve five physical quantities viz, u, v, s, t and a and in each equation, there are 4 quantities If any 3 of these 4 quantities are given, we can find the fourth quantity by using the above equations of motion. In solving any problem, we should, therefore, write down the given quantities at the beginning and then try to find the values of the required quantities by using one or more equations of motion. The choice of the appropriate formula or formulae depends on the given values of the physical quantities in the problem.

4.6. Solved Examples

Ex. 1 If in the rectilinear motion of a point, the time t and the displacement from a given point satisfy the equation

$$t=ax^2+bx+c,$$

where a, b, c are given constants, prove that

(i) the velocity at time t is $\frac{1}{2ax+b}$.

and (ii) the acceleration is inversely proportional to the cube of the distance from a certain fixed point in the line of motion and find the distance of this fixed point from the given point.

(i) Differentiating both sides of the equation

$t=ax^2+bx+c$

with respect to t, we get

$$1=2ax.\frac{dx}{dt}+b\frac{dx}{dt}$$

$$=\frac{dx}{dt}(2ax+b)$$

$$\Rightarrow\frac{dx}{dt}=\frac{1}{2ax+b}. \qquad ...(1)$$

Hence, the velocity at times t is $\frac{1}{2ax+b}$.

(iii) Differentiating both sides of (1) with respect to t again, we get

$$\frac{d^2x}{dt^2}=\frac{-2a}{(2ax+b)^2}\cdot\frac{dx}{dt}$$

$$=\frac{-2a}{(2ax+b)^2}\times\frac{1}{2ax+b}\ \text{(from 1)}$$

$$=\frac{-2a}{(2ax+b)^3}=\frac{-1}{4a^2\left(x+\frac{b}{2a}\right)^3}$$

$$=\frac{k}{x'^3}, \qquad ...(2)$$

where $k=\frac{-1}{4a^2}$=constant and $x'=x+\frac{b}{2a}$ so that x' representst he distance of the point at time t from the fixed point whose distance from the starting point as the origin is $x=\frac{-b}{2a}$.

From (2), it follows that $a\propto\frac{1}{x'^3}$. Hence the result.

Remark : In the above problem, we see that the acceleration is not constant so that we cannot use the equations of motion of section 4.2. In the case of variable acceleration, we must use calculus as shown above. We shall consider more problems of this type below :

Ex. 2. A straight rod AB slides on a plane with its ends on two fixed perpendicular lines OX and OY. Show that the speeds of the extremities of the rod are inversely proportional to their distances from O.

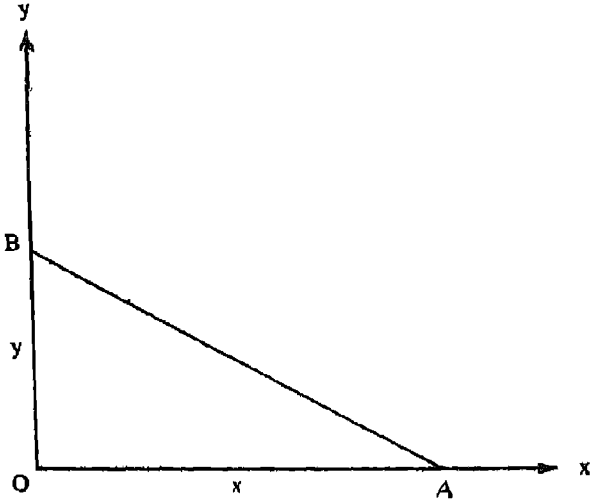


Fig. 25

Let A and B be at distances x and y respectively from O, and let $AB=a$.

Then $x^2+y^2=a^2$.

Differentiating both sides with respect to t, we get

$$x\frac{dx}{dt}+y\frac{dy}{dt}=0.$$

$$\Rightarrow xu+yv=0,$$

where u and v are the velocities of A and B respectively.

$\therefore \quad xu+yv=k,$ say, where k is a constant.

$$\therefore \quad u=\frac{k}{x}$$

and $v = \frac{-k}{y}$

so that the speeds of A and B are inversely proportional to their respective distances from O

Ex. 3. The acceleration of a moving body varies directly as the time which has elapsed since its velocity was 10 m/s. If it travels 168 m in 6 s following this instant, find how far it will travel in the next 4 s.

Let v be the velocity of the body at any time t measured from the instant when the velocity was 10 m/s.

Then $\frac{dv}{dt} = a = kt$ where k is a constant

Integrating both sides with respect to t, we get

$$v = k\int t\,dt = \frac{kt^2}{2} + c, \qquad \ldots(1)$$

where c is a constant of integration

Now, initially $v = 10$ when $t = 0$.

This gives $10 = c$.

Hence, from (1)

$$v = \frac{1}{2}kt^2 + 10.$$

To find the distance s, we use the formula

$$\frac{ds}{dt} = v.$$

Hence,

$$\frac{ds}{dt} = \frac{1}{2}kt^2 + 20.$$

Integrating both sides with respect to t, we get

$$s = \frac{1}{2}k\int t^2 dt + 10\int dt,$$

$$= \frac{1}{6}kt^3 + 10t + c', \qquad (2)$$

where c' is a constant of integration.

Now, initially, $s = 0$ when $t = 0$.

Hence, $c' = 0$.

Thus, we have from (2),

$$= \frac{1}{6}kt^3 + 10t \qquad \ldots(3)$$

Now, to find the unknown constant k, we use the given condition that $s=168$ when $t=6$

Hence, putting these values of s and t in (3), we get

$$168=\frac{1}{6}k\,6^3+10\times 6$$

$$\Rightarrow\quad 36k=168-60=108$$

$$\Rightarrow\quad k=\frac{108}{36}=3.$$

Thus from (3), we get

$$s=\frac{1}{2}t^3+10\,t \qquad \cdot\ (4)$$

Now, to find the distance in the next 4 seconds i.e. from $t=6$ to $t=6+4=10$, we substitute $t=10$ in (4) and get

$$s_{10}=\frac{1}{2}\times 10^3+10\times 10=500+100+600.$$

Also $s_6=168$.

Hence, the body moves a distance of $(600-168)$ m

$=432$ m during the next 4 seconds.

Ex. 4. In what time would a body acquire a velocity of 270 m/h if it was started with a velocity of 2 cm/s and moved with an acceleration of $\frac{1}{2}$ cm/s^2 ? Find the average velocity of the body for its motion during this period in two ways by using (i) the initial and the final velocities of the body and (ii) the initial velocity, the acceleration and the time.

Given that

$u=2$ cm/s, $\quad v=270$ m/h

$$=\frac{27000}{60\times 60}\text{ cm/s}$$

$$=\frac{15}{2}\text{ cm/s}$$

and $a=\frac{1}{2}$cm/s^2.

To find t.

Using the formula

$v=u+at$, we have

$$\frac{15}{2}=2+\frac{1}{2}\,t\Rightarrow t=11.$$

Hence, the required time is 11 s.

(i) The average velocity

$$V=\frac{u+v}{2}=\frac{1}{2}\left(2+\frac{15}{2}\right)$$
$$=\frac{19}{4}.$$

Hence, the required average velocity

$$=\frac{19}{4}\text{ m/s}.$$

(ii) We know that the average velocity is equal to the velocity at the middle of the interval of time i.e. the velocity at $\frac{1}{2}\,t$ s $=\frac{11}{2}$ s is the required average velocity.

Hence,

$$V=u+\frac{1}{2}at$$
$$=2+\frac{11}{4}=\frac{19}{4}\ .$$

Hence, we get the same value for the average velocity.

Ex 5. A train travelling at 30 km/h is brought to rest uniformly at a station in $1\frac{1}{2}$ minutes. At what distance from the station were the brakes applied ? What was the retardation in m/s^2 ?

Here $u=30\ \text{km/h}=\frac{30000}{60\times60}\text{ m/s}=\frac{25}{3}\text{ m/s}$ and

$$t=\frac{3}{2}\text{ min }=\frac{3}{2}\times60\text{ s}=90\text{ s}.$$

When the train is brought to rest, then $v=0$. Here we shall find a. Now, using the formula

$v=u+at$, we get

$$0=\frac{25}{3}+90a$$

$$\Rightarrow a=\frac{-25}{3\times 90}=\frac{-5}{54}.$$

The negative sign indicates that the train moved with retardation.

Hence, the required retardation of the train is $\frac{5}{54}$ m/s^2.

To find the distance traversed in 90 s, we use the formula

$$s=ut+\frac{1}{2}at^2.$$

Hence, substituting the values of u, a and t in the above formula, we get

$$s=\frac{25}{3}\times 90-\frac{1}{2}\times\frac{5}{54}\times 90\times 90$$
$$=750-375=375.$$

Hence, the required distance=375 m.

Ex. 6. A particle starting with a given velocity moves for 3 s with constant acceleration, during which time it describes 81 m ; the acceleration then ceases and during the next 3 s, it describes 72 m. Find its initial velocity and acceleration.

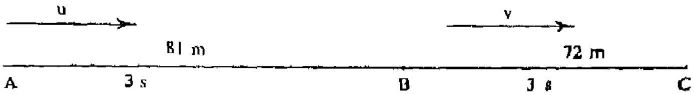


Fig 26

Let the distance $AB=81$ m be described by the particle in 3 s moving with initial velocity u m/s and acceleration a m/s^2 and let the distance $BC=72$ m be described in 3 s by the particle moving with uniform velocity v m/s attained at B For the motion from A to B, we have $s=81$ m and $t=3$ s. Hence, using the formula

$$s=ut+\frac{1}{2}at^2,$$ we get

$$81=3u+\frac{9a}{2} \qquad \ldots(1$$

Again, the particle describes a distance of 72 m from B to C with a uniform velocity V in 3 s. Hence, $V=\frac{72}{3}$ m/s=24 m/s. Considering the motion of the particle from A to B again and using the formula $v=u+at$, we get

$$24=u+3a \qquad \dots(2)$$

From (1) and (2), we shall calculate u and a. Now, from (2), we have

$$u=24-3a \qquad \dots(3)$$

Substituting in (1), we get

$$81=72-9a+\frac{9a}{2}$$

$$\Rightarrow 9=-\frac{9a}{2} \Rightarrow a=-2.$$

Hence, the particle moves with a retardation of 2 m/s². Putting $a=-2$ in (3), we get

$$u=24+3\times 2=30.$$

Hence, the initial velocity of the particle is 30 m/s.

Ex. 7. Two particles start at the same instant from a point A and move in the same direction along a straight line AB. The first has a uniform velocity of 40 m/s while the second starts with an initial velocity of 16 cm/s and has a uniform acceleration of 6 cm/s². Find the time that elapses before the two particles meet again.

Let the two particles meet together at C after t s.

Fig 27

Now, for the first particle,

$$AC=40\,t \qquad \dots(1)$$

For the second particle,

$$u=16 \text{ cm/s}, \qquad a=6 \text{ cm/s}^2.$$

Hence, $AC=16\,t+\frac{1}{2}\times 6\,t^2$

$$=16\,t+3\,t^2 \qquad \dots(2)$$

Hence, from (1) and (2), we get

$$40\,t=16\,t+3\,t^2$$

$$\Rightarrow 3\,t^2=24\,t$$

$$\Rightarrow t=0 \text{ or } 8.$$

At $t=0$, the two particles meet initially. After this, they meet again after 8 seconds.

Ex. 8. A particle moves with uniform acceleration If a, b, c, be the distances described by it in pth, qth and rth seconds respectively, show that

$$a(q-r)+b(r-p)+c(p-q)=0. \qquad \text{[Allahabad 10]}$$

Let f be the uniform acceleration and u be the initial velocity. Then S_p=distance described in pth second

$$=u+\frac{1}{2}f(2p-1)$$

$$\Rightarrow a=u+\frac{1}{2}f(2p-1) \qquad \ldots(1)$$

Similarly,

$$b=u+\frac{1}{2}f(2q-1) \qquad .(2$$

and $$c=u+\frac{1}{2}f(2r-1) \qquad ..(3)$$

Multiplying (1), (2) and (3) by $q-r$, $r-p$ and $p-q$ respectively and then adding, we get

$$a(q-r)+b(r-p)+c(p-q)$$

$$=u(q-r+r-p+p-q)$$

$$+\frac{1}{2}f\Big((2p-1)(q-r)+(2q-1)(r-p)+(2r-1)(p-q)\Big)$$

$$=0+\frac{1}{2}f(2pq-2pr-q+r+2qr-2qp-r+p+2rp-2rq-p+q)$$

$$=0+0=0$$

Hence the result.

Ex. 9. Two trains on the same line are approaching one another with velocities u_1 and u_2 respectively. When there is a distance x between them, each is seen from the other. Prove that it is just possible to avoid a collison if

$$u_1^2 f_2+u_2^2 f_1=2f_1 f_2 x,$$

where f_1 and f_2 are the greatest retardations which the brakes can produce in the respective trains.

Let $AB=x$ so that when the two trains come at A and B respectively moving in opposite directions, they apply brakes to avoid collison As a result, both the trains will move with the greatest retardations f_1, f_2, which the brakes can produce The two trains will just avoid the collision when their final velocities, v_1 and v_2, become zero at their meeting point, say, C. Let $AC=S_1$ and $BC=S_2$ so that $x=S_1+S_2$.

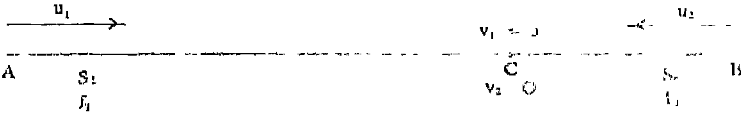


Fig. 28

Now, considering the motion of the first train and using the formula $v^2=u^2+2as$, we get

$$0=u_1^2-2f_1S_1$$

$$\Rightarrow \quad S_1=\frac{u_1^2}{2f_1}.$$

Similarly, considering the motion of the second train, we get

$$S_2=\frac{u_2^2}{2f_2}$$

$$\text{Hence,} \quad S_1+S_2=\frac{u_1^2}{2f_1}+\frac{u_2^2}{2f_2}$$

$$\Rightarrow \quad x=\frac{f_2u_1^2+f_1u_2^2}{2f_1f_2}$$

$$\Rightarrow \quad u_1^2f_2+u_2^2f_1=2f_1f_2x$$

Hence the result.

Ex 10. A train moves 5 km from rest to rest in 10 minutes. The greatest speed is 50 km/h and the acceleration and retardation are both uniform. Find the distance travelled at full speed.

Here there are three kinds of motion of the train : (i) the train first moves with a uniform acceleration, say a_1 m/s^2 from A to B during time, say, t_1s, (ii) then it moves for t s with a uniform maximum velocity, say,

V m/s attained at B and finally (iii) it moves for t_2s with a uni retardation say, a_2 m/s^2, from C to D where its velocity becomes zero

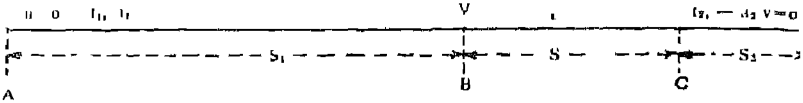


Fig 29

Let $AB=S_1$m, $BC=S$m and $CD=S_2$m

so that $S_1+S+S_2=5000$

and $t_1+t+t_2=600$

Here the maximum velocity

$$=V=\frac{53\times 1000}{60\times 60}\text{m/s}=\frac{125}{9}\text{m/s}$$

Now considering the motion of the train from A to B and using formulae

$v=u+at$ and $S=ut+\frac{1}{2}at^2$ successively, we get

$$\frac{125}{9}=a_1t_1$$

and $S_1=\frac{1}{2}a_1t_1^2$

From (3) and (4), eliminating a_1, we get

$$S_1=\frac{1}{2}\cdot\frac{125}{9}\cdot t_1=\frac{125}{18}\,t_1$$

For the motion of the train from B to C,

$$S=Vt=\frac{125}{9}\,t$$

For the motion from C to D, we have, by using the formulae

$v=u+at$ and $S=ut+\frac{1}{2}at^2$,

$$0=V-a_2t_2$$

$$\Rightarrow \quad \frac{125}{9}=a_2t_2 \qquad \ldots(7)$$

and $$S_2=\frac{125}{9}t_2-\frac{1}{2}a_2t_2^2 \qquad .(8)$$

Eliminating a_2 from (7) and (8), we get

$$S_2=\frac{125}{9}t_2-\frac{1}{2}\cdot\frac{125}{9}t_2=\frac{125}{18}t_2 \qquad \ldots(9)$$

Substituting the values of S_1, S and S_2 from (5), (6) and (9) in (1), we get

$$\frac{125}{18}t_1+\frac{125}{9}t+\frac{125}{18}t_2=5000$$

$$\Rightarrow \quad t_1+t_2+2t=720 \qquad \ldots(10)$$

Subtracting (2) from (10), we get

$$t=120.$$

Hence, the distance travelled at the maximum speed $V=Vt$ is

$$120\times\frac{125}{9}\text{ m}=\frac{5000}{3}\text{ m}=1.667\text{ km}.$$

Ex. 11. A point moves in a straight line with a constant acceleration. If the distances of the moving point from a fixed point on the line be x_1, x_2, x_3 at the instants t_1, t_2, t_3, prove that the acceleration is

$$2\left\{\frac{(x_2-x_3)\,t_1+(x_3-x_1)\,t_2+(x_1-x_2)\,t_3}{(t_2-t_3)\,(t_3-t_1)\,(t_1-t_2)}\right\} \qquad [\text{ U. P. 46 }]$$

Let O be a fixed point on the straight line and let A, B, C be three points on the line at distances x_1, x_2 and x_3 from O. Let the point start

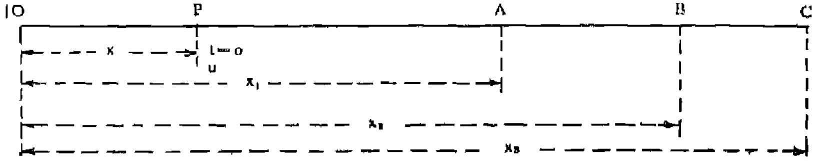


Fig. 30

from P at a distance x from O with initial velocity u so that at the instant $t=0$, the velocity of the point is x. Let a be the uniform acceleration. Then from the from formula $S=ut+\frac{1}{2}at^2$, we get

$$x_1-x=ut_1+\frac{1}{2}at_1^2 \qquad \ldots(1)$$

$$x_2-x=ut_2+\frac{1}{2}at_2^2 \qquad (2)$$

$$x_3-x=ut_3+\frac{1}{2}at_3^2 \qquad (3)$$

Subtracting (3) from (2), we get

$$x_2-x_3=u\,(t_2-t_3)+\frac{1}{2}\,a\left(t_2^2-t_3^2\right) \qquad (4)$$

Subtracting (1) from (3), we get

$$x_3-x_1=u\,(t_3-t_1)+\frac{1}{2}\,a\left(t_3^2-t_1^2\right) \qquad (5)$$

Subtracting (2) from (1), we get

$$x_1-x_2=u\,(t_1-t_2)+\frac{1}{2}\,a\left(t_1^2-t_2^2\right) \qquad (6)$$

Multiplying (4), (5) and (6) by t_1, t_2 and t_3 respectively and adding, we get

$$\begin{aligned}
&(x_2-x_3)\,t_1+(x_3-x_1)\,t_2+(x_1-x_2)\,t_3\\
&=u\,(t_1t_2-t_1t_3+t_2t_3-t_1t_2+t_3t_1-t_2t_3)\\
&\quad+\frac{1}{2}a\left[t_1\left(t_2^2-t_3^2\right)+t_2\left(t_3^2-t_1^2\right)+t_3\left(t_1^2-t_2^2\right)\right]\\
&=\frac{1}{2}\,a\left[-t_1^2\,(t_2-t_3)+t_1\,(t_2-t_3)\,(t_2+t_3)-t_2t_3\,(t_2-t_3)\right]\\
&=\frac{1}{2}\,a\,(t_2-t_3)(t_1t_2+t_1t_3-t_1^2-t_2t_3)\\
&=\frac{1}{2}\,a\,(t_2-t_3)\,[(-t_1\,(t_1-t_2)+t_3\,(t_1-t_2)]\\
&=\frac{1}{2}\,a\,(t_2-t_3)\,(t_3-t_1)\,(t_1-t_2)
\end{aligned}$$

Hence, $a=\dfrac{(x_2-x_3)\,t_1+(x_3-x_1)\,t_2+(x_1-x_2)\,t_3}{(t_2-t_3)\,(t_3-t_1)\,(t_1-t_2)}$.

Ex 12. A constable seeing a thief at a distance x m starts with a velocity u and moves with an acceleration α in order to catch him, while the thief runs with an acceleration β, starting from rest. Show that the constable will overtake the thief either if $\alpha \geqslant \beta$ or if $\alpha \leqslant \beta \leqslant \alpha+\frac{u^2}{2x}$.

Let C and T be the initial positions of the constable and the thief respectively so that $CT=x$. Let the constable catch the thief at O in time t

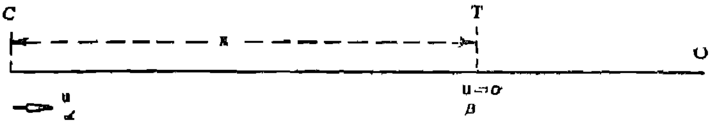


Fig 31

Now, $\quad CO=CT+TO \qquad (1)$

But CO is the distance traversed by the constable in time t and TO is that traversed by the constable in the same time.

$$CO=ut+\frac{1}{2}\alpha t^2 \text{ and}$$

$$TO=\frac{1}{2}\beta t^2.$$

Hence, from (1),

$$ut+\frac{1}{2}\alpha t^2=x+\frac{1}{2}\beta t^2$$

$$\Rightarrow \quad (\beta-\alpha)t^2-2ut+2x=0$$

$$\Rightarrow \quad t=\frac{2u+\sqrt{4u^2-8x(\beta-\alpha)}}{2(\beta-\alpha)}$$

$$=\frac{u\pm\sqrt{u^2-2x(\beta-\alpha)}}{\beta-\alpha}.$$

If the constable catches the thief, the two values of t must be real. Hence,

$$u^2-2x(\beta-\alpha)\geqslant 0$$

i.e. either $\quad \alpha\geqslant\beta \qquad (i)$

or, $\quad \alpha\leqslant\beta$ and $u^2\geqslant 2x(\beta-\alpha)$

i.e. $\quad \alpha\leqslant\beta$ and $\frac{u^2}{2x}+\alpha\geqslant\beta$

$\Rightarrow \quad \alpha\leqslant\beta\leqslant\alpha+\frac{u^2}{2x} \qquad (ii)$

It may be noted that in the first case (*i*), one value of *t* is positive and hence this value of *t* gives the required time. In the second case (*ii*), both the values are positive. Of these two values, the smaller gives the time when the constable first catches the thief.

Ex. 13 Two points *P* and *Q* move in a straight line *AB* The point *P* starts from *A* in the direction *AB* with velocity u_1 and acceleration f_1 and at the same time, *Q* starts from *B* in the direction *BA* with velocity u_2 and acceleration f_2. If they pass one another at the middle point of *AB* and arrive at the end of *AB* with equal velocity, prove that

$$(u_1+u_2)(f_1-f_2)=8(f_1u_2-f_2u_1).$$

Let $AB=x$ and t be the time taken by each point to reach the middle point *O* Let *V* be the common velocity of *P* and *Q* at *B* and *A* respectively

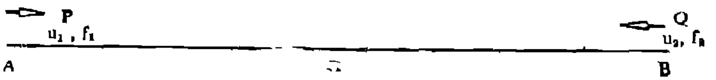


Fig. 32

Now, considering the motion of the point *P* from *A* to *B* and using the formula

$v^2=u^2+2as$, we get

$$v^2=u_1^2+2f_1x \tag{1}$$

Similarly, considering the motion of the point *Q* from *B* to *A*, we have

$$V^2=u_2^2+2f_2x \tag{2}$$

From (1) and (2), we have

$$u_1^2+2f_1x=u_2^2+2f_2x$$

$$\Rightarrow 2(f_1-f_2)x=u_2^2-u_1^2 \tag{3}$$

Now, the points *P* and *Q* reach the middle point *O* in time *t*. Hence, using the formula $S=ut+\frac{1}{2}at^2$, we get

$$\frac{x}{2}=u_1\,t+\frac{1}{2}f_1\,t^2 \qquad (4)$$

and $$\frac{x}{2}=u_2\,t+\frac{1}{2}f_2\,t^2 \qquad (5)$$

Subtracting (5) from (4), we get

$$0=(u_1-u_2)\,t+\frac{1}{2}(f_1-f_2)\,t^2$$

$$\Rightarrow \qquad t=\frac{2\,(u_2-u_1)}{f_1-f_2} \qquad [\because \quad t\neq 0]$$

Putting this value of t in (4), we get

$$\frac{x}{2}=\frac{2u_1(u_2-u_1)}{f_1-f_2}+\frac{1}{2}f_1.\frac{4(u_2-u_1)^2}{(f_1-f_2)^2}$$

$$\Rightarrow \qquad (f_1-f_2)^2\,x=4\,u_1\,(u_2-u_1)\,(f_1-f_2)+4\,f_1\,(u_2-u_1)^2$$

$$=4\,(u_2-u_1)\,(u_1f_1-u_1f_2+f_1u_2-u_1f_1)$$

$$=4\,(u_2-u_1)\,(f_1u_2-f_2u_1)$$

$$\Rightarrow \qquad \frac{(f_1-f_2)\,(u_2^2-u_1^2)}{2}=4\,(u_2-u_1)\,(f_1u_2-f_2u_1) \qquad [\text{from } (3)]$$

$$\Rightarrow \qquad (u_2+u_1)\,(f_1-f_2)=8\,(f_1u_2-f_2u_1).$$

Hence the result.

Ex. 14. Three screens are placed at a distance a apart, and a bullet fired through them passes them at times t_1, t_2 and t_3 recorded by an electric chronograph. If the retardation is assumed uniform, show that its magnitude is

$$\frac{2\,(t_3-t_2+t_1)\,a}{(t_3-t_1)\,(t_3-t_2)\,(t_2-t_1)}.$$

Let u, v and w be the velocities of the bullet when it strikes the first, cond and third screens respectively at times t_1, t_2 and t_3 respectively. t f be the uniform retardation throughout the motion of the bullet. bullet strikes the second screen after time t_2-t_1 from the instant when ikes the first screen Similarly, the bullet strikes the third screen after t t_3-t_2 from the instant when it strikes the second screen. Now, cdering the motion of the bullet from the first screen to the second aing the formulae

$$=ut-\frac{1}{2}ft^2 \text{ and } v=u-ft, \text{ we get}$$

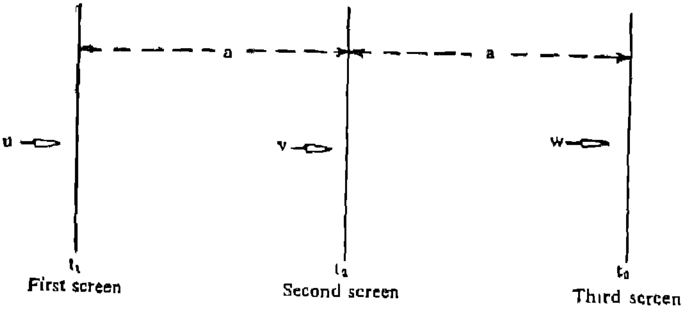


Fig. 33

$$a=u(t_2-t_1)-\frac{1}{2}f(t_2-t_1)^2 \quad \cdots(1)$$

$$v=u+f(t_2-t_1) \quad (2)$$

Similarly, considering the motion of the bullet from the second screen to the third screen, we get

$$a=v(t_3-t_2)-\frac{1}{2}f(t_3-t_2)^2 \quad \ldots(3)$$

Substituting v from (2) in (3), we get

$$a=u(t_3-t_2)-f(t_2-t_1)(t_3-t_2)-\frac{1}{2}f(t_3-t_2)^2 \quad (4)$$

Now, from (1) we get

$$u=\frac{a+\frac{1}{2}f(t_2-t_1)^2}{t_2-t_1}$$

Substituting in (4), we get

$$a(t_2-t_1)=a(t_3-t_2)+\frac{1}{2}f(t_3-t_2)(t_2-t_1)^2$$
$$-f(t_2-t_1)^2(t_3-t_2)-\frac{1}{2}f(t_3-t_2)^2(t_2-t_1)$$

$$\Rightarrow \quad 2a(t_3-2t_2+t_1)=-f(t_3-t_2)(t_2-t_1)^2+2f(t_3-t_2)(t_2-t_1)^2$$
$$+f(t_3-t_2)^2(t_2-t_1)$$
$$=f(t_3-t_2)(t_2-t_1)(t_3-t_2+2t_2-2t_1-t_2+t$$
$$=f(t_3-t_2)(t_2-t_1)(t_3-t_1)$$

Hence,

$$f=\frac{2a(t_3-2t_2+t_1)}{(t_3-t_1)(t_3-t_2)(t_2-t_1)}$$

Hence the result.

EXERCISE 4

1 The distance d cm travelled by a particle in t s is given by

$$d=20t^2+50t+19.$$

Find the velocity and the acceleration after 3s.

[Ans. 170 cm/s ; 40 cm/s^2]

2. If $d=63t-6t^2-t^3$, where d is in metre and t in second, find the velocity after 2s and the distance travelled before the particle stops. [C U. 58]

[Ans. 27 m/s ; 108 m]

3 A particle moves along a straight line, and its distance from a fixed point on the line after t s from start is given by $x=a+bt+ct^2$, where a, b, c are constants. Prove that it moves with a constant acceleration.

4. If the relation between v cm/s and s cm is given by $v=10+\frac{s}{15}$, find the acceleration when $s=900$ cm. [Ans. $4\frac{2}{3}$ cm/s^2]

5. Find the acceleration of a body accelerated uniformly if its speed changes from (a) 10 m/s to 25 m/s in 5s (b) 20 m/s in 3s.

[Ans. (a) 5 m/s^2 (b) -5 m/s^2]

6. A body moving with a speed of 36 km/h is brought to rest in 10 s. What is the negative acceleration and the distance travelled by the body before coming to rest ?

[Ans. 1 m/s^2 ; 50 m]

7 A body has a velocity of 15 m/s at a certain instant and 10 s later it has a velocity of 45 m/s. If the velocity changes uniformly, find the space described [Ans. 300 m].

8. A particle starting from rest moves with an acceleration of 4 cm/s^2. Find in what time it will acquire a velocity of 60 cm/s. Also find the distance described in this time.

[Ans. 15 s ; 450 cm]

9 A body starts from rest and at the end of 10 s, it is moving at the rate of 22 m/s ; 5 seconds later, its velocity is 105.6 km/h. Is the acceleration constant ? [Ans No]

10 A French electric train gets up a speed of 20 km/h in 30 s and a certain English train a speed of 57.6 km/h in 1 minute. Which train gets up speed more rapidly ? [Ans. English train]

11. A body whose velocity is being constantly accelerated has at a certain instant a velocity of 22 m/s and in the following minute, it travels a distance of 10320 m. Find the acceleration.

[Ans. 5 m/s^2]

12. A bullet fired into a target loses half its velocity after penetrating 3 cm. How much further will it penetrate. [Ans 1 cm]

13. A train which is moving at the rate of 36 km/h is brought to rest in 2 minutes with a uniform retardation. Find the retardation and also the distance that the train travels before coming to rest. $\left[\text{Ans. } \frac{1}{12} \text{ m/s}^2 \text{ ; } 600 \text{ m}\right]$

14 A body has an initial velocity of 200 cm/s and is subject to a retardation of 2 cm/s^2 At what time will its velocity be zero and how far will it then travel ? [Ans 1 minute 40 seconds]

15. A particle describes 650 m in 10s and its velocity at the end of that time is 80 m/s. Find the initial velocity and the acceleration. [Ans. 50 m/s ; 3 m/s^2]

16. The velocity of a train increases from 15 km/h to 60 km/h while it moves through a distance of $\frac{1}{8}$ km If the acceleration is uniform, find its magnitude. $\left[\text{Ans. } \frac{25}{24} \text{ m/s}^2\right]$

17. A particle moving in a straight line covers 12 m, 18 m and 42 m in successive intervals of 3s, 2s and 3s. Prove that these distances are consistent with the supposition that the point is moving with a uniform acceleration. Find also this uniform acceleration [Ans. 2 m/s^2]

18. A point moving with a uniform acceleration describes 25 m in the half second which elapses after the first second of its motion and 198 m in the eleventh second of its motion Prove that the initial velocity and the acceleration of the point are respectively 30 m/s and 16 m/s^2

19 A ball rolling down a slope with a uniform acceleration passes three posts driven in the ground at equal distances. The velocities when passing the three successive posts are x, y, z. Prove that x^2, y^2, z^2 are in A.P.

20. A particle starts with an initial velocity u and passes successively over two-halves of a given distance with accelerations f and f' respectively. Show that the velocity is the same as if the whole distance were traversed with a uniform acceleration $\frac{1}{2}(f+f')$

[C.U. 40]

21. An aeroplane in taking off from an airport has a run of 2000 m. If it leaves the run-way in 10s from the start and moving with a

uniform acceleration, what is (i) its acceleration, (ii) the velocity with which it takes off the ground ?

[Ans. (i) 40 m/s^2 (ii) 400 m/s]

22. A bullet moving at 100 m/s is fired into a wood block in which it penetrates 50 cm. If the same bullet moving with the same velocity were fired into a similar piece of wood but only 12.5 cm thick, with what velocity would it emerge supposing the resistance of wood in both the cases to be the same and uniform ?

[Ans. 86.6 m/s]

23. A locomotive driver rushing at 72 km/h finds a signal 110 m ahead of him indicating he should stop He instantly applies brakes to stop the train. The train retards uniformly and stops 10 m before the signal post. What time did he take to stop the train ? [Ans 10s]

24 A train which is moving with a uniform retardation is observed to take 20s and 30s to travel successive kilometres. How much further will it travel before coming to rest ? $\left[\text{Ans. } 408\frac{1}{3}\text{ m}\right]$

25. A train takes 50s in going from a station A to a station B and in going from B to a station C also it takes 50s. The distance between A and B is $\frac{1}{2}$ km and that from B to C is $\frac{3}{4}$ km. If the acceleration be uniform, find its magnitude Find also the velocities at A, B and C.

[Ans. .1 m/s^3 ; 7.5 m/s ; 12.5 m/s ; 17.5 m/s]

26. A train is travelling at 50 km/h when the steam is turned off and the brakes are applied. If the speed is reduced to 40 km/h in 10s, find how far the train will travel after the brakes are applied before coming to rest. $\left[\text{Ans. } \frac{25}{72}\text{ km}\right]$

27. A lizard, at an initial distance of 21 cm behind and moving from rest with an acceleration of 2 cm/s^2 pursues an insect which is crawling uniformly along a straight line at 20 cm/s. Find when and where the lizard will catch the insect.

[Ans. 21s ; at 441 cm from the starting point]

28. A train running with acceleration passes by two stations A and B with velocities u and v. Is the velocity of the train at half-time equal to, greater than or less than the velocity half-way ?

[Ans. Less]

29. A train is travelling at 36 km/h when the driver observes a stop-

signal. After 2 seconds, he applies the brakes which produce a constant braking force and bring the train to rest in 4 seconds.
(i) Determine the distance travelled by the train from the time the driver first sees the obstruction until the train comes to rest
(ii) Had the train been travelling at 72 km/h, how much longer would it have taken to stop, had the same action been taken by the driver ?
(iii) What total distance would the train have travelled in the second case ? [Ans. (i) 40 m (ii) 4s (iii) 120 m]

30. A car travelling with a uniform acceleration has a velocity of 18 km/h at the instant of observation and 54 km/h after covering a distance of 400 m. How much further must it travel to attain a velocity of 72 km/h ? [Ans. 350 m]

31. Two bodies start from rest from the same point and move in the same straight line with uniform but different accelerations. After 5 minutes one has a velocity of 2 m/s more than the other How much is the first body in advance of the other at this instant ? [Ans. 300 m]

32. An object starts from rest and accelerates uniformly at 2 m/s^2 for 10 seconds. It then moves with a constant speed (attained at the end of 10 s) for the next 20 seconds, and is then brought to rest under uniform retardation while travelling a further distance of 200 metres. Calculate
(i) the total distance travelled and
(ii) the total time of journey. [Ans. 700 m ; 50 s]

33. A car *A* is travelling at 36 km/h along a straight road The driver of this car notices another car *B*, 100 metres ahead of it, travelling at 72 km/h in the same direction. *A* is then accelerated uniformly and overtakes *B*, after *B* has travelled 400 metres.
(i) What was the velocity of car *A* at the instant it overtakes *B* and (ii) what acceleration was imparted to *A* in order to overtake *B* ? [Ans. (i) 40 m/s (ii) 1.5 m/s^2]

34. A train travels from a station *X* to a station *Y* in 45 minutes. At a point *Z*, somewhere between *X* and *Y*, it attains a maximum velocity of 45 km/h. If it travels with uniform acceleration from *X* to *Z* and uniform retardation from *Z* to *Y*, find the distance between *X* and *Y*, it being supposed that the train starts from rest at *X* and comes to rest at *Y* $\left[\text{Ans. } 16\frac{7}{8}\text{ km}\right]$

35. A train starts from *A* and stops at *B*. The velocity increases uniformly till it reaches a maximum velocity *V* and then it

decreases uniformly. Show that the time taken by the train to run from A to B is $\frac{2x}{V}$, where x is the distance between the two stations.

36 One body A lies 30 m in front of another body B, and both start from rest. If A moves off with uniform acceleration of 4 m/s^2 and B runs with a uniform velocity of 10 m/s, is it possible for B to overtake A ? What happens if B runs with a uniform velocity of 16 m/s ?

[Ans No In the second case B will meet with A twice during their motion]

37. A car starting from rest at A travels to B with uniform acceleration of $\frac{22}{45}$ m/s^2 ; from B to C its speed is constant ; at C, the brakes are applied, the subsequent retardation being uniform and equal to $\frac{11}{45}$ m/s^2. If the total distance travelled is 8580 m and the total time is $5\frac{1}{2}$ minutes, find the time taken by the car to travel from B to C. [Ans. 1 minute]

38. A car A is travelling on a straight level road with uniform speed of 60 km/h It is followed by another car B moving with a speed of 70 km/h. When the distance between them is 2 5 km, the car B is given a retardation of 20 km/h^2. After what distance and time will B catch up A ? [I.I.T. 66]

[Ans. .5 h, 32.5 km]

39 The two ends of a train moving with a constant acceleration pass a certain point with speeds u and v respectively. With what velocity does the centre of the train pass the same point ?

$\left[\text{Ans. } \sqrt{\frac{1}{2}(u^2+v^2)}\right]$

40. A cyclist riding 18 km/h passes a car just as it begins to move in the same direction. The car maintains an acceleration of $\frac{1}{2}$ m/s^2 for 18 seconds and then moves uniformly. How far will it have run before overtaking the cyclist ?

[Ans. 101.25 m]

41. A point moving with uniform acceleration describes in the last second of its motion $\frac{9}{25}$th of the whole distance. If it started

from rest, how long was it in motion and through what distance did it move if it described 10 cm in the first second ?

[Ans. 5 s, 250 cm]

42. A particle starts with a velocity of 200 cm/s and moves in a straight line with a retardation of 10 cm/s^2. Find the time when it has described 1500 cm and explain the double answers.

[Ans. 10 s and 30 s]

43. A burglar's car had a start with an acceleration of 2 m/s^2 A police vigilant party came in a car to the spot at a velocity of 20 m/s after 5 s and continued to chase the burglar's car with a uniform velocity. Find the time in which the police van will overtake the car. [A.M.I.E.]

[Ans. 5 s]

44. A bus starts from rest with an acceleration of $2a$ m/s^2. Show that a man who can walk at the rate of $8a$ m/s cannot catch the bus if he is more than $16a$ m behind it.

45 A particle is projected in a straight line with a certain velocity and a constant acceleration. One second later, another particle is projected after it with half the velocity and double the acceleration. When the second particle overtakes the first, their velocities are 31 m/s and 22 m/s respectively. Prove that the distance traversed is 48 m.

46. A train passes another train on a parallel track. The first is running at a uniform speed of 24 km/h and the second is running at a speed of 6 km/h with an acceleration of 50 cm/s^2 How long will it be before the second train overtakes the first again, and how far will the train run in the interval ?

$$\left[\text{Ans. } 20 \text{ s}, \ 133\tfrac{1}{3} \text{ m}\right]$$

47. A car accelerates from rest at a constant rate a for some time after which it decelerates (i e. moves with retardation) at a constant rate b to come to rest. If the total time lapse is t seconds, evaluate (i) the maximum velocity reached and (ii) the total distance travelled [I I.T.]

$$\left[\text{Ans. (i) } \frac{abt}{a+b};\ \text{(ii)} \frac{abt^2}{2(a+b)}\right]$$

48. A particle starts from a point O with a uniform velocity of 1.5 m/s and after 2 s, another particle leaves O in the same direction with a velocity of 37.5 cm/s and with an acceleration equal to 37.5 cm/s^2 Find when and where it will overtake the first particle. [Ans. 8 s after its start and at a distance of 15 m]

49. A train starts from A with a uniform acceleration of 20 cm/s^2. After 2 minutes, the train attains the full speed and moves uniformly for 10 minutes. It is then brought to rest at B by the brakes producing a constant retardation of 24 cm/s^2. Find the distance AB. [Ans. 17.04 km]

50. A particle traverses a distance of 450 m in a straight line at an average speed of 10 m/s starting from rest and finishing at rest. It moves with uniform acceleration for the first 10 s and is brought to rest by a uniform retardation in the last 20 s of its motion, and moves at a uniform speed during the rest of its motion. Find the acceleration and retardation.

[Ans. 1 5 m/s^2 ; .75 m/s^2]

51 A train stopping at two stations 2 km apart takes 4 minutes on the journey from one of the stations to the other. Assuming that its motion is first that of uniform acceleration x and then that of uniform retardation y, prove that

$\frac{1}{x}+\frac{1}{y}=4$, a kilometre and a minute being the units of distance and time respectively.

52 The distance x of a particle moving in a straight line from a fixed point O on the line at time t is given by

$x=a\cos(nt+k)$, where a, n and k are constants. Show that its acceleration varies as x and is directed towards O.

53. A particle moves along a straight line and at a distance x from a fixed point O on the line, its velocity is $\mu\sqrt{\frac{c-x}{x}}$. Prove that its acceleration is directed towards O and is inversely proportional to the square of the distance.

54. The law of motion of a body moving along a straight line is $x=\frac{1}{2}vt$, x being its distance from a fixed point on the line at time t, and v, its velocity there Prove that it moves with a constant acceleration.

55. A point moves along a straight line in such a way that after t s, its acceleration is $(t+2)$ cm/s^2. If the point starts from rest, find its velocity after 3 s and the distance travelled in that time. $\left[\text{Ans. } 10\frac{1}{2} \text{ cm/s} ; 13\frac{1}{2} \text{ cm.}\right]$

56. A tramcar starts from rest with an acceleration which is initially 1 m/s^2. The acceleration varies directly as the time and it

decreases until it vanishes after 15 s. Find the velocity after 15 s and the total distance travelled in these 15 s.

[Ans. 7.5 m/s ; 75 m]

57. If a point moves along a straight line in such a way that when its distance from a fixed point is s m, its velocity is $(50-2\,s)$ m/s. Find the acceleration (i) when it has moved a distance of 15 m and (ii) when its velocity is 10 m/s.

[Ans (i)—40 m/s^2 ; (ii)—20 m/s^2]

58 The acceleration of a body starting from rest is initially 22 m/s^2 and decreases uniformly with the distance covered until it reaches zero, when the body has travelled 550 m, after which the velocity is constant. Find its speed when it has covered (i) 100 m, (ii) 550 m and the distance it has travelled when its speed is 88 m/s. [Ans (i) 63.2 m/s ; (ii) 0 m/s ; 220 m]

59. Five minutes before the finish, a cyclist had a velocity of 27 km/h. At the finish it was 36 km/h. What is his acceleration in m/s^2 assuming it to be constant ? [Ans. .5 m/s^2]

60. A body covers half of its journey with a speed of a m/s and the other half with a speed of 1 m/s. Calculate the average speed of the body during the whole journey. $\left[\text{Ans. } \frac{2\,ab}{a+b} \text{ m/s}\right]$

61. A body moves from a place A to a place B at a constant speed of 20 km/h and returns immediately at a constant speed of 30 km/h. Explain why the average speed of the body during the whole journey is 24 km/h and not 25 km/h.

62. Find the distance described during the tenth second by a particle having initial velocity of 60 m/s and moving with a uniform retardation of 2 m/s^2 Find the time in which it comes to rest and the distance described before it comes to rest.

[Ans. 41 m ; 30 s ; 900 m]

63. A body moving with a uniform acceleration describes 55 m in the sixth second from rest. How much distance will it move in the eighth second ? [Ans. 75 m]

64. A body travels 200 cm in the first two seconds and 220 cm in the next four seconds. Assuming the motion to be under uniform acceleration throughout, calculate the velocity at the end of seventh second from the start. [I.I.T. 64]

65. A point is moving with uniform acceleration. In the eleventh and fifteenth seconds from the commencement, it moves through 720 cm and 960 cm respectively Find its initial velocity and the acceleration with which it moves. [Ans. 90 cm/s ; 60 cm/s^2]

66. In two successive seconds, a particle moves through 20.5 cm and 23.5 cm respectively. Assuming that it was moving with uniform acceleration, find its velocity at the commencement of the first of these two successive seconds and its acceleration. Find how far it had moved from rest before the commencement of the first second. [Ans. 19 cm/s, 3 cm/s^2; $60\frac{1}{6}$ cm]

67. A point moves in a straight line and describes 16 cm in the 2nd second of its motion, 28 cm in the 5th second and 52 cm in the 11th second. Prove that these distances are consistent with the supposition that the motion is uniformly accelerated. Also find the whole distance described in 10 s from the beginning of the motion. [Ans. 300 cm]

68. A point traverses 7 m in the first second, 11 m in the third second and 17 m in the sixth second Is the acceleration uniform ? [Ans. Yes]

69. Two cars start off for a race with velocities u m/s and v m/s and their accelerations are α m/s^2 and β m/s^2 respectively. If the race ends in the dead beat, prove that the length of the race is

$$\frac{2\,(u-v)\,(u\beta-v\alpha)}{(\alpha-\beta)^2},$$

70. In a car race, two cars C_1 and C_2 start simultaneously from rest. C_1 travels throughout with a uniform acceleration a_1 and C_2 with a_2. If C_1 wins the race by time t, show that the velocity of C_1 at the finish was $t\sqrt{a_1a_2}$ more than that of C_2, when C_2 reached the finish point.

71. Two cars left a point A simultaneously and reached a point B also simultaneously after 2 hours. The first car travelled half of the distance at an average speed of 30 km/h and the other half of the distance at 45 km/h. The second car travelled the entire distance with a constant acceleration.

(i) At what instant were the speeds of both the cars the same ?

(ii) Will one of them overtake the other en-route ?

[Ans. After 50 minutes or 75 minutes ; no car overtakes the other en-route]

72. An express train is overtaking a goods train on the same line, their velocities being u_1 and u_2 respectively. When there is a distance x between them, each is seen from the other. Prove that it is just possible to avoid a collision if $(u_1-u_2)^2=2(f_1+f_2)\,x$, where f_1 is the greatest retardation and f_2, the greatest

acceleration which can be produced in the two trains respectively

73. A goods train is travelling at a speed of v_2. It is followed by a passenger train travelling on the same track with a speed, v_1 ($v_1 > v_2$). When the distance between the two trains is x, the driver of the passenger train applies brakes and gives his train a uniform retardation a Show that (i) there will be no collision if (i) $x > \frac{(v_1 - v_2)^2}{2a}$ and (ii) there will be a collision if $x < \frac{(v_1 - v_2)^2}{2a}$

74 A particle starts from rest in a straight line with alternate acceleration and retardation of magnitudes 2 m/s² and 1 m/s² respectively during equal intervals of 15 s. Find the distance travelled in 1 minute. [Ans 1575 m]

75. A particle starting from rest moves with constant acceleration, x m/s² for 10 s ; travels with constant velocity for a further 10 s and then retards at $2x$ m/s² to come to rest at a place 350 m from its starting point Find the value of x. [Ans. 2]

76. The distance between two stations X and Y is 7.875 km. A train starting from rest at X accelerates uniformly for 6 minutes at the end of which its velocity is 45 km/h. The train runs at this constant speed until it decelerates uniformly for 3 minutes to stop at a station Find the total time for the journey.

[Ans. 15 minutes]

77. Astronauts are trained for highly accelerated motions. An astronaut drives a rocket-driven sled on a straight horizontal track and attains a speed of 1620 km/h in 1 8 s, starting from rest.

(i) Assuming the acceleration f to be constant, compare it with the acceleration due to gravity, g, where $g = 10$ m/s²

(ii) What is the distance travelled in these 1 8 s ?

[Ans. (i) $\frac{f}{g} = 25$ (ii) 405 m]

78. At the instant the traffic light turns green, a car starts from rest with a constant acceleration of 2 m/s². At the same instant, a truck travelling with a constant speed of 10 m/s, overtakes and passes over the car. (i) How far beyond the starting point will the car overtake the truck ? (ii) How fast will the car be travelling at that instant ? [Ans. (i) 100 m ; (ii) 20 m/s]

79. In a particle acceleration, an α-particle enters a 2 m long tube at 1000 m/s and leaves at 9000 m/s. How long was the particle in the tube and what was its acceelration ?

[Ans. 400 micro seconds ; 2×10^7 m/s²]

80 A car A is travelling at 36 km/h along a straight road. The driver of this car notices another car B, 100 m ahead of it, travelling at 72 km/h in the same direction. A is then accelerated uniformly and it overtakes B after B has travelled through 400 m.
(i) What was the velocity of car A at the instant it overtakes B?
(ii) What acceleration was imparted to A in order to overtake B?
[Ans. (i) 40 m/s ; (ii) 1.5 m/s^2]

81. A charged particle enters an electric field with a speed of 5×10^6 m/s and comes to rest in 10 cm. What is the retardation due to the electric field? [Ans. 1.25×10^{14} m/s^2]

82. In an electron-gun of a TV tube, an electron enters the accelerating region at 10^4m/s and emerges out at 4×10^6m/s. If the acceleration of the electron was 8×10^{14}m/s^2, what is the length of the accelerating path? [Ans. 1 cm]

83 In a car accident, the skid marks on the road had a length of 9m. Assuming that the maximum retardation would not exceed the acceleration due to gravity, g, where $g=10$ m/s^2, was the driver exceeding the speed limit of 36 km/h? [Ans. Yes]

84. An arrow while being shot from a bow was accelerated and left the bow over a distance of 60 cm. If the arrow left the bow with a speed of 60 m/s, what was the average acceleration imparted by the bow? [Ans 300 m/s]

85. A body moves in a straight line AB and its distance from A after t seconds is d. If d and t satisfy the relation

$$d= 25\, t + 375\, t^2$$

for all values of t, find (i) the average velocity of the body during the 4th second and (ii) the velocity at the end of 4th second.
[Ans. (i) 2.875 m/s ; (ii) 3.25 m/s]

86. A train travels from rest at one station to rest at another station in the same straight line distant d m. It moves for the first part of the distance with an acceleration of a m/s^2 and for the remainder with a retardation of b m/s^2. Show that it will accomplish the journey in $\sqrt{\dfrac{2\,(a+b)\,d}{ab}}$ seconds

87. If v_1, v_2, v_3 be the average velocities in three successive intervals of time t_1, t_2, t_3 of a point moving in a straight line with uniform acceleration, show that

$$\frac{v_1-v_2}{v_2-v_3}=\frac{t_1+t_2}{t_2+t_3}.$$

88. A particle starts from rest and moves with uniform acceleration for a time t_1. The acceleration then ceases and it moves with uniform velocity for time t_2 describing a distance a. It then moves with uniform retardation and comes to rest after a time t_3. Prove that the whole space described is

$$a\left(1+\frac{t_1+t_3}{2t_2}\right).$$

89. Two particles move in the same straight line starting at the same moment from the same point in it, the first moves with constant velocity u and the second with constant acceleration f, during the time that elapses before the second catches the first. Show that the greatest distance between the particles is $\frac{u^2}{2f}$ at the end of time $\frac{u}{f}$ from rest.

90. A train starting from rest travels the first part of its journey with constant acceleration f, the second part with constant speed v and the third part with constant retardation f'. The train is then brought to rest again. If the average speed for the whole journey is $\frac{7}{8}v$, show that the train is travelling at constant speed for three-quarters of the total time. Find also what fraction of the distance is described with constant speed.

[Ans. $\frac{6}{7}$th]

91. A point moving with constant acceleration from A to B in the straight line AB has speeds u, v at A, B respectively. Find the speed at C, the mid-point of AB, and show that if the time from A to C be twice that from C to B, then $v=7u$.

$\left[\text{Ans } \sqrt{\frac{u^2+v^2}{2}}\right]$

92. Two particles, P and Q, start together and move from the same point A along the same line AB. P has a uniform velocity of 20 m/s and Q, a uniform acceleration of 4 m/s^2 and no initial velocity. Find when and where they meet again. Before they meet again, find when the distance between the two will be maximum and what is the maximum distance ?

[Ans. After 10 s from start at a distance 200 m ;
5 s from start ; 50 m]

93. A point moves in a straight line from A to B with uniform acceleration. Show that if P is the position at half-time,

$$\frac{AP}{PB}=\frac{3u+v}{u+3v},$$

where u, v are the initial and final velocities.

94. Prove that for a particle moving with uniform acceleration f in a straight line,

$$f=\frac{2}{t+t'}\left(\frac{x'}{t'}-\frac{x}{t}\right),$$

where x is the space described in t s and x' during the next t' s.

[C.U 48 ; P.U. 43]

95. A point moving in a straight line with uniform acceleration describes distances a and b in successive intervals of time t_1 and t_2. Prove that

(i) the acceleration is $\dfrac{2\ (bt_1-at_2)}{t_1t_2\ (t_1+t_2)}$

and (ii) $\dfrac{1}{t_1}-\dfrac{1}{t_2}+\dfrac{1}{t_3}=\dfrac{3}{t_1+t_2+t_3}$,

where t_1, t_2, t_3 are the times when successive equal distances are described.

96. The two ends of a train moving with a constant acceleration pass a certain point with velocities u and v. Find, in terms of u and v, what proportion of length of the train will have passed the point after a time equal to half that taken by the train to pass it.

[Ans. $\dfrac{3u+v}{4\ (u+v)}$]

97. Two particles move in the same straight line with constant accelerations f_1 and f_2. If their velocities be u_1 and u_2 at a certain instant when they are at distances y_1 and y_2 from a fixed point on the line, prove that they can not pass each other more than twice, and if they do so twice, the interval between the two times of passing is

$$\frac{2}{f_1-f_2}\sqrt{(u_1-u_2)^2-2(y_1-y_2)\ (f_1-f_2)}\ .$$

98. A particle starting from rest moves in a straight line, first with uniform acceleration a and then with uniform retardation b. If it comes to rest in time t, measured from the beginning, after having described a space x, prove that

$$t^2=2x\left(\frac{1}{a}+\frac{1}{b}\right).$$

[C.U. 45]

99 The velocity of a bullet, v m/s, is reduced to zero by penetrating x m inside some sand If it penetrates y m $(y<x)$, the velocity is reduced to u m/s. Show that $u : v=\sqrt{x-y} : \sqrt{x}$

100. A particle starts from rest with acceleration f ; at the end of time t, it becomes $2f$, at the end of time $2t$, $3f$ and so on. Find the velocity at the end of time nt and show that the distance described is

$$\frac{n(n+1)(2n+1)}{12}ft^2. \qquad \left[\text{Ans } \frac{n(n+1)ft}{2}\right]$$

101. A body starts from rest at A and moves with uniform acceleration f in a straight line. T s later, a second body starts from A and moves with uniform velocity u in the same line. If $u>2fT$, prove that the second body will be ahead of the first for a time

$$\frac{2}{f}\sqrt{u(u-2fT)}.$$

102. A distance x is divided into n equal parts at the end of each of which the acceleration of a moving particle is increased by $\frac{f}{n}$. Show that the velocity of the particle after describing the distance is

$$\sqrt{fx\left(3-\frac{1}{n}\right)},$$

where f is the initial acceleration of the particle starting from rest

103. The velocity of a train increases at a constant rate f_1 from 0 to V, then remains constant for an interval, and finally decreases to 0 at the constant rate f_2. If x be the total distance described, prove that the total time taken is

$$\frac{x}{v}+\frac{v}{2}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$$

104. Four stations A, B, C, D on a railway are so placed that $AB=\frac{1}{2}BC=CD=20$ km. A train has a maximum speed of 50 km/h to which it can accelerate uniformly in 10 minutes and from which it can decelerate uniformly to rest in 5 minutes Find the shortest time in which the train can travel from start at A to stop at D, (a) stopping for 2 minutes at B and at C, (b) without stopping at B or C

[Ans. (a) 2 hours $2\frac{1}{2}$ minutes ; (b) 1 hour $43\frac{1}{2}$ minutes]

105 Two particles move in straight lines over equal distances from rest to rest. The first particle has constant acceleration f for the first third of the time, constant speed for the second third, and constant retardation f for the last third of the time. The second particle has constant acceleration f for the first third of the distance, constant speed for the second third, and constant retardation f for the last third of the distance Show that the times taken by the two particles are in the ratio $3\sqrt{3} : 5$.

106. A particle is projected from O along a straight line and moves with uniform retardation to rest at a point A The distance OA is divided into n equal intervals. Show that the ratio of the time taken to describe the rth interval to the time taken to describe OA is

$$\frac{\sqrt{n-r+1}-\sqrt{n-r}}{\sqrt{n}}.$$

107. A train starting from rest accelerates uniformly till it attains a speed of 72 km/h. It maintains this speed and passes a 23.6 km mark in 20 minutes from the commencement of motion Find its acceleration in m/s^2. [Ans. 0.5 m/s^2]

108. Prove that it is impossible for a particle to move in a straight line so that its velocity varies as the distance described from the commencement of the motion.

Chapter 5

Motion in a Straight Line under Gravity

5 1 Acceleration due to Gravity

When a body is dropped from some height above the earth's surface, it falls to the earth due to the attractive force exerted on it by the earth. This attractive force is called *"gravity"* The direction in which this force acts is approximately the line joining the centre of the earth and the centre of the mass of the body. This force is nearly constant at a particular part of the earth's surface and is independent of the mass or shape of the body at the place It may be observed that as the body falls from a height above the earth's surface, its velocity continually increases as it comes nearer the earth's surface. In other words, it falls with an acceleration. It has been shown by many experiments that if the body is free from air resistance, the acceleration is uniform and acts in the direction of the gravity. Thus if a body is let fall in vacuum, it will fall with a constant acceleration along a straight line joining the body and the centre of the earth i.e. vertically downwards This acceleration is called "the acceleration due to gravity". It is generally denoted by the letter '*g*' The value of '*g*' is the same for all bodies at a particular place, but its value varies slightly from one place to another on the surface of the earth Its value is 9.78 m/s^2 at the equator and 9.83 m/s^2 at the poles. In all numerical examples, we shall always take the value of '*g*' as 9 8 m/s^2 unless it is stated otherwise in the problems. For more approximate calculation, we sometimes take its value as 10 m/s^2.

When a body is thrown vertically upwards with certain initial velocity, then this velocity will gradually decrease till it becomes zero. After this, the body will start coming back to the earth with acceleration "*g*". Thus when the body moves up vertically, it moves with negative acceleration, $-g$, and when it moves down, it moves with acceleration, g. Hence, g is taken as positive for the downward motion and negative for the upward motion of a body.

5.2. Equations for Vertical Motion

Case 1 : Motion of a body projected vertically downwards

Let a body be projected vertically downwards from a point O with velocity u. The body moves downwards in which g is positive. To study the motion, we can take the point O as the origin and the vertical through O as the x-axis. The downward direction is taken as the positive direction. The time t is measured from O. Let v be the velocity of the body after time t and let the distance covered by the body be h. Let h_t be the distance covered in tth second. Then using the equations of motion of a body on a straight line, discussed in the previous chapter, we get

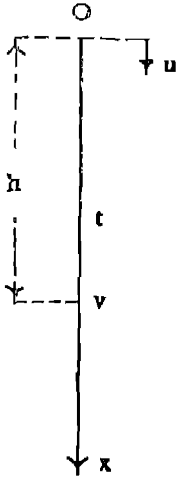


Fig. 34

$$v=u+gt,$$

$$h=ut+\frac{1}{2}gt^2,$$

$$v^2=u^2+2gh$$

and $$h_t=u+\frac{1}{2}g\,(2t-1),$$

which are the required equations of motion of a body projected with an initial velocity u vertically downwards.

In particular, if the body falls freely from a height h, then the initial velocity u will be zero and the above equations of motion will reduce to

$$v=gt,$$

$$h=\frac{1}{2}gt^2,$$

$$v^2=2gh$$

and $$h_t=\frac{1}{2}g\,(2t-1).$$

Case 2 : Motion of a body projected vertically upwards

Let a body be projected vertically upwards from a point O with a velocity u. The body moves upwards in which g is negative with decreasing velocity. To study the motion, we take O as the origin and the vertical through O in the upward direction as the positive direction of the x-axis We measure time from O. Let v be the velocity of the body after time t and let the distance covered in time t be h. Let h_t denote the distance covered in t th second In the upward direction, we take u, v, h to be positive but g is negative. Hence, the equations of motion become

$$v=u-gt,$$

$$h=ut-\frac{1}{2}gt^2,$$

$$v^2=u^2-2gh$$

and $$h_t=u-\frac{1}{2}g(2t-1)$$

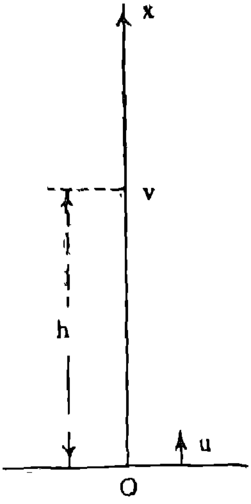


Fig. 35

5.3 Convention of Signs

The acceleration due to gravity, g, is taken to be positive while considering the motion of a body in the vertically downward direction and negative in the case of upward motion of the body. For determining the signs of u, v and h, we can choose any direction, upward or downward, as positive. For example, if we choose the vertically downward direction as positive and if a body is dropped from a rising balloon, rising up with a velocity u, then u should be taken as negative while considering the downward motion of the body.

It should be noted carefully that h in the above equation of motion denotes the magnitude of the "displacement" and not the "total distance" covered by the body. Hence, h is the distance between the initial and the final positions of the body. It is positive or negative according as the displacement is in the positive or negative direction We consider an example. Let a body be thrown vertically upwards with an initial velocity u from a certain height above the earth's surface. Let the body rise at a height h_1 above the point of projection, where its velocity becomes zero. The body then comes back to the ground. Let h_2 be the greatest height above the ground. Now, if we consider the downward motion of the body, we can choose the downward direction as positive. Then u should be taken as negative, and g and v positive. The value of h should be taken as h_2-h_1 and *not* h_1+h_2

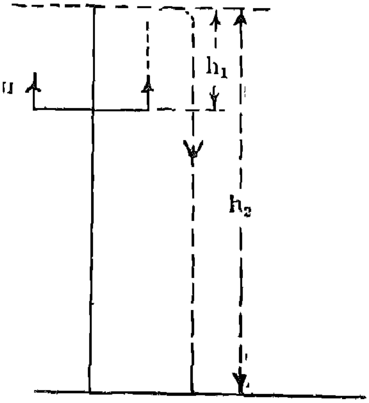


Fig. 36

5.4. Greatest Height and the Time to attain the Greatest Height

A body is thrown vertically upwards with an initial velocity u. To find the greatest height attained by the particle and the time to attain the greatest height.

The body will attain the greatest height H when its final velocity will be zero. Considering the vertically upward motion and using the formula $v^2=u^2-2gh$, we get

$$0=u^2-2gH$$

$$\Rightarrow \quad H=\frac{u^2}{2g}.$$

This formula gives us the greatest height attained by the body. To find the time T taken by the body to attain the greatest height, we use the formula $v=u-gt$ and obtain

$$0=u-gT$$

$$\Rightarrow \quad T=\frac{u}{g}.$$

This formula gives us the time taken by the body to attain the greatest height i.e the time of ascent of the body.

After attaining the greatest height, the body will begin to fall. If T' be the time of fall and v be the velocity on reaching the ground again, then considering the downard direction as positive, we get

$$H=\frac{1}{2}gT'^2$$

$$\Rightarrow \quad \frac{u^2}{2g}=\frac{1}{2}gT'^2 \qquad \Rightarrow \quad T'^2=\frac{u^2}{g^2}$$

$$\Rightarrow \quad T'=\frac{u}{g}=T$$

and $$v^2=2gH=2g\frac{u^2}{2g}=u^2$$

$$\Rightarrow \quad v=u.$$

Thus for a body projected vertically upwards

(*i*) *the time of rise=time of fall*

and (*ii*) *the velocity on reaching the ground again =the initial velocity of protection*

Hence, the total time of flight$=\frac{2u}{g}$.

5.5 Time and Velocity at any Height

Let a body be projected vertically upwards from O with an initial velocity u and at any time t, let the particle be at P at a height h above the ground so that $OP=h$.

Then taking the upward direction as positive, we have

$$h=ut-\frac{1}{2}gt^2$$

$$\Rightarrow \quad \frac{1}{2}gt^2-ut+h=0$$

$$\Rightarrow \quad t=\frac{u\pm\sqrt{u^2-2gh}}{g}$$

$$=\frac{u}{g}\pm\frac{\sqrt{u^2-2gh}}{g}.$$

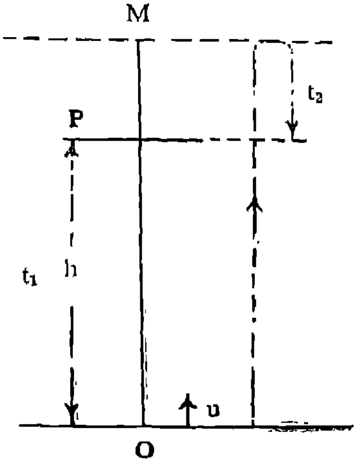


Fig. 37

If $u^2-2gh>0 \Rightarrow u^2>2gh$, then both the values of t are real The reason for this two real values of t is that the body is at the same height twice, once on its way up, the time t_1 for which is $t_1=\frac{u}{g}-\frac{\sqrt{u^2-2gh}}{g}$ and once on its way down for which the time t_2 is given by $t_2=\frac{u}{g}+\frac{\sqrt{u^2-2gh}}{g}$. Obviously, $t_2>t_1$.

Again, the two values of t are imaginary if $u^2<2gh$ i e. $u<\sqrt{2gh}$ It means that the body can not rise to the height h if its initial velocity u is less than $\sqrt{2gh}$

In case $u^2=2gh$, then $t=\frac{u}{g}$ which gives us the time taken by the body to attain the greatest height. Hence, in this case OP will be the greatest height.

We also note that when $u^2>2gh$,

$$t_1=\frac{u}{g}-\frac{\sqrt{u^2-2gh}}{g}=\text{time to attain a height } OP$$

and $t_2=\frac{u}{g}+\frac{\sqrt{u^2-2gh}}{g}=$ time to attain a height $OM+$time to describe a distance MP

But the time to attain the greatest height $OM=\frac{u}{g}$.

Hence, it follows that the time to reach M from P during the upward motion$=\frac{u}{g}-t_1=\frac{\sqrt{u^2-2gh}}{g}$ and the time to reach P from M during the downward motion$=t_2-\frac{u}{g}=\frac{\sqrt{u^2-2gh}}{g}$.

Thus, *time of rise to the point of maximum height M from any point P=time of fall from the point of maximum height M to the same point.*

To find the velocity at any height h, we use the formula $v^2=u^2-2gh$. This gives $v=\pm\sqrt{u^2-2gh}$. The positive sign gives the velocity of the body on its way up through the point, and the negative sign gives that on its way down. Thus *at the same point of its path, the magnitude of the velocity of a body is the same when falling as when rising.* A particular case of this result is that when reaching the point of projection again during the downward motion of a body, the velocity is the same as the velocity of projection

To find the interval between these equal velocities, we note that this interval is the difference of times t_1 and t_2 taken by the body to move up and down the distance PM. Thus the interval between these equal

velocities$=t_2-t_1=\frac{2\sqrt{u^2-2gh}}{g}=\frac{2v}{g}$,

where v is the velocity at P.

5 6 Motion along a Smooth Inclined Plane

Let AB be the vertical section of a smooth inclined plane inclined at an angle α to the horizon, and let P be a body on the plane. The line AB represents the line of greatest slope. Now, by parallelogram of vectors, the vertical acceleration g is equivalent to an acceleration $g \cos\alpha$ perpendicular to the plane in the downward direction and an acceleration $g \sin\alpha$ down the plane. Now, the body has no motion perpendicular to the plane and so no motion is created due to the acceleration $g \cos\alpha$ perpendicular to the plane. Hence, the body moves down the plane with an acceleration $g \sin\alpha$. If we consider the motion up the plane, then the acceleration up the plane should be taken as $-g \sin\alpha$. The investigation of motion of a body along the inclined plane is similar to that along a vertical line except that we substitute $g \sin\alpha$ instead of g for motion down the plane and $-g \sin\alpha$ instead of $-g$ for motion up the plane in the equation of vertical motion. Thus the equation of motion down a smooth plane of inclination α are

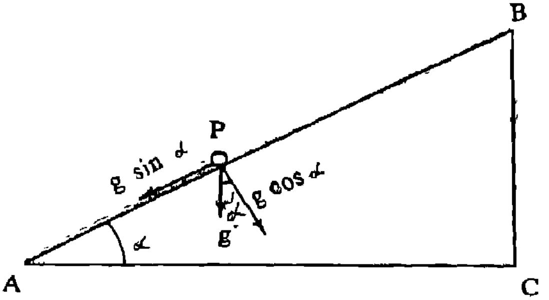


Fig. 38

$$v = u + g \sin\alpha \,.\, t,$$

$$s = ut + \frac{1}{2} g \sin\alpha \,.\, t^2,$$

$$v^2 = u^2 + 2g \sin\alpha \quad s \text{ and}$$

$$s_t = u + \frac{1}{2} g \sin\alpha \,.\, (2t-1).$$

If the body starts from rest, then u should be taken as zero in all these formulae.

Similarly, the equations of motion of a body up a smooth inclined plane are given by the following formulae :

$$v=u-g\sin\alpha\,.\,t,$$
$$s=ut-\frac{1}{2}g\sin\alpha\;t^2,$$
$$v^2=u^2-2g\sin\alpha\,.\,s\text{ and}$$
$$s_t=u-\frac{1}{2}g\sin\alpha\,.\,(2t-1).$$

We note that if h be the height of the plane, then $\sin\alpha=\frac{h}{s}$ so that the formulae

$$v^2=u^2\pm 2g\sin\alpha.s\text{ become}$$
$$v^2=u^2\pm 2gh$$

which are independent of α, the inclination of the plane. Hence, the final velocity v for motion of the body up or down the plane will be the same for all planes of the same height, provided the initial velocity u remains the same in each case.

5.7. Motion down a Smooth Chord of a Vertical Circle

We shall now prove a well-known theorem on the motion of a body down a smooth chord of a vertical circle. The theorem can be stated as follows :

The time taken by a body to slide down any smooth chord of a vertical circle starting from rest, the chord being drawn from the highest point of the circle is constant and is equal to the time taken to fall down its vertical diameter.

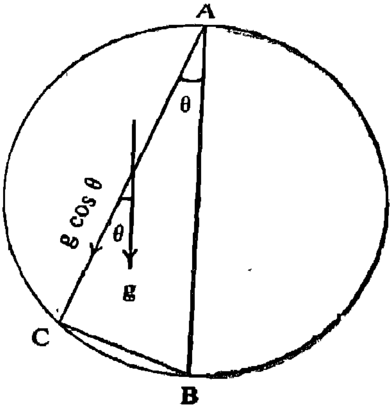


Fig. 39

Let AC be any chord drawn through the highest point A of a vertical circle and let AB be the vertical diameter. Let $\angle BAC=\theta$ and $AB=d$ Let T be the time taken by the body to reach C starting from A. Now, the acceleration along $AB=g\cos\theta$.

Also, $AC=d\cos\theta$

$$[\quad \angle ABC=90°]$$

Hence, $d\cos\theta=\frac{1}{2}g\cos\theta.T^2$

$$\Rightarrow\quad T=\sqrt{\frac{2d}{g}}$$

which is independent of θ and is, therefore, constant. This time is the same as that taken by a body to fall from rest down the vertical diameter AB.

Remark : It can be shown similarly that the times of sliding down from rest along all chords of a vertical circle ending at the lowest point are equal.

5.8 Solved Examples

Ex 1 A body is thrown vertically upwards and rises to a height of 10 m. Calculate (a) the velocity with which the body was thrown upwards and (b) the time taken by the body to reach the highest point.

Here the maxium height is 10 m. Hence, the velocity of the body at this height is zero Here $h=10$ and $v=0$

Let u be the initial velocity.

(a) Considering the upward motion as positive and using the formula $v^2=u^2-2gh$, we get

$$0=u^2-2\times 9.8\times 10$$

$$\Rightarrow \quad u^2=196 \quad \Rightarrow \quad u=14.$$

Hence, the required initial velocity of projection=14m/s

(b) In this case, using the formula

$$v=u-gt, \text{ we get}$$

$$0=14-9.8t$$

$$\Rightarrow \quad t=\frac{14}{9.8}=\frac{10}{7}.$$

Hence, the required time taken by the body to reach the greatest height$=\frac{10}{7}$ s.

Ex. 2. A particle is projected upwards from the ground and after some time it is seen at a height of 21 m falling downwards with a velocity of 16m/s. How long before this was it moving upwards through the same point and what was its velocity then ? Find also the time from start to the highest point (Take $g=10$ m/s^2)

Let the particle be projected from O on the ground with initial velocity u. When it comes at P at a height of 21m, let the upward velocity at P be v m/s. The particle then attains the maximum height OA and so the velocity of the particle at A is zero The particle then comes back from A and when it comes to the same point P again, its downward velocity is

16m/s. Let t seconds be the time taken by the particle to describe the distances PA and AP. Clearly, the displacement during t seconds is 0. Taking the upward velocity from P as positive, and using the formula $h=ut-\frac{1}{2}\cdot gt^2$, we get

$$0=vt-\frac{1}{2}\times 10\,t^2 \Rightarrow 5t=v \qquad ..(1)$$

Again, using the formula $v=u-gt$, and considering the motion for the distances PA and AP, we get

$$-16=v-10t \Rightarrow 10t=v+16 \qquad .(2)$$

From (1) and (2), we get

$$10t=5t+16 \Rightarrow 5t=16$$

$$\Rightarrow t=\frac{16}{5}=3.2$$

Putting $t=\frac{16}{5}$ in (1), we get

$$v=5\times\frac{16}{5}=16$$

Hence, the particle was moving for 3.2 seconds to describe the distances PA and AP and its velocity at P while moving up is 16m/s.

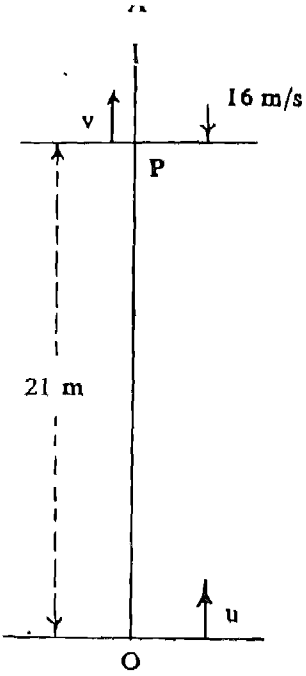


Fig. 40

Now, to find the time taken by the particle to reach the highest point, we should find the initial velocity u at the ground. Now, considering the upward motion from O to P and using the formula $v^2=u^2-2gh$, we get

$$16^2=u^2-2\times 10\times 21$$

$$\Rightarrow u=\sqrt{676}=26.$$

Hence, the required time from the start to the highest point

$$=\frac{u}{g}=\frac{26}{10}S=2.5\ S.$$

Ex. 3. A body falls freely from the top of a tower and falls $\frac{4}{5}$ th of the whole distance during the last section of its flight. Find the height of the tower (Take $g=10\text{m/s}^2$).

Let t s be the time of fall. Then the distance traversed in t th second

$$=\frac{1}{2}g\,(2t-1)=5(2t-1)$$

Now, the whole distance$=\frac{1}{2}gt^2=\frac{1}{2}\times 10t^2=5t^2$

Hence, $\frac{4}{5}$ th of this distance$=5t^2\times\frac{4}{5}=4t^2$

$\therefore\ 4t^2=5\ (2t-1)$

$\Rightarrow 4t^2-10t+5=0$

$\Rightarrow t=\frac{10\pm\sqrt{100-80}}{8}=\frac{10\pm\sqrt{20}}{8}=\frac{10\pm2\sqrt{5}}{8}=\frac{5\pm\sqrt{5}}{4}$

Since t should be greater than 1, we reject $t=\frac{5-\sqrt{5}}{4}$ which is less than 1.

Hence, $t=\frac{5+\sqrt{5}}{4}$

Hence, the required height of the tower

$$=\frac{1}{2}gt^2=\frac{1}{2}\times 10\times\left(\frac{5+\sqrt{5}}{4}\right)^2 \text{ m}$$

$$=\frac{5(25+5+10\sqrt{5}}{16} \text{ m}$$

$$=\frac{75+25\sqrt{5}}{8} \text{ m}$$

Ex. 4. A balloon has been ascending vertically at a uniform rate for 4.5 s and a stone let fall from it reaches the ground in 7s. Find the velocity of the balloon, and its height whence the stone is let fall.

The balloon is moving up at a uniform velocity. Let um/s be the uniform velocity of the balloon and let h m be the height of the balloon during 4 5 s.

$\therefore\ h=4\ 5\ u$ (1)

At this height h, a stone is dropped from the balloon. Initially, the velocity of the stone will be equal to the velocity of the balloon i.e u m/s in the upward direction The stone takes 7s to reach the ground i.e to traverse a distance h. Considering the dowhward motion as positive, we get by using the formula $h=ut+\frac{1}{2}gt^2$,

$$h=-7u+\frac{1}{2}g\times 7^2$$

$$=-7u+5\times 49=245-7u \qquad \text{.. (2)}$$

From (1) and (2), we get

$$4.5\,u=245-7u$$

$$\Rightarrow 11.5\;u=245$$

$$\Rightarrow u=\frac{2450}{115}=\frac{490}{23}\simeq 21.30$$

$$\therefore \text{ From (1), } h=245-\frac{7\times 490}{23}=\frac{2205}{23}\simeq 95.67$$

Hence, the velocity of the balloon is 21.30 m/s and its height whence the stone is let fall is 95.61 m nearly.

Ex. 5. A particle is dropped from a height of 144 m and after falling $\frac{2}{3}$rd of the distance passes another particle which was projected upwards from the ground at the instant when the first was dropped. Find to what height the second particle rises.

Let the first particle be dropped from c and after t seconds, let it come to B, where $CB=\frac{2}{3}\times 144$ m$=96$ m. Let the secon particle be projected from A on the ground and after the same time t, it comes to B where $AB=(144-96)$ m$=48$ m. Now, considering the downward motion of the first particle from C to B and using the formula $h=ut+\frac{1}{2}gt^2$, we get

$$96=\frac{1}{2}gt^2 \qquad (1)$$

$$\Rightarrow t=\sqrt{\frac{192}{g}} \qquad (2)$$

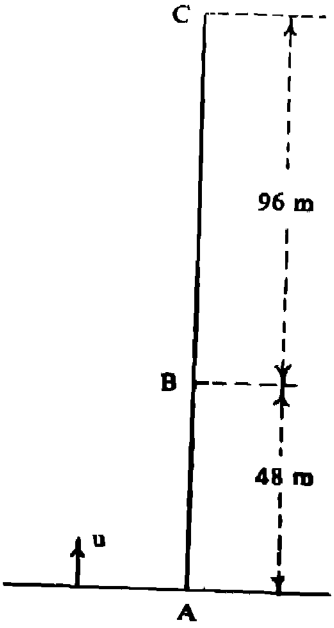


Fig. 41

Now, considering the upward motion of the second particle from A to B and using the formula $h=ut-\frac{1}{2}gt^2$, we get

$$48=ut-\frac{1}{2}gt^2$$

$$=u\sqrt{\frac{192}{g}}-96 \qquad \text{[from (1) and (2)]}$$

$$\Rightarrow u=\sqrt{\frac{g}{192}}\times 144$$

If H be the greatest height attained by the second particle, then

$$H=\frac{u^2}{2g}=\frac{g\times 144^2}{192}\times\frac{1}{2g}$$

$$=\frac{144\times 144}{192\times 2}=54.$$

Hence, the required height attained by the second particle is 54 m

Ex.6 . A particle is projected upwards with a velocity u ; n seconds afterwards, another particle is projected upwards with a velocity v from the same point. If the two particles meet at the highest point reached by the first, show that

$$v-u=\frac{g^2n^2}{2\,(u-ng)}\,.$$

The greatest height attained by the first particle is $H=\frac{u^2}{2g}$ and the time to attain this height is

$$t=\frac{u}{g}.$$

The second particle attains this height in time $\left(\frac{u}{g}-n\right)$ seconds.

Hence, considering the upward motion of the second particle and using the formula $h=ut-\frac{1}{2}gt^2$, we get

$$\frac{u^2}{2g}=v\left(\frac{u}{g}-n\right)-\frac{1}{2}g\left(\frac{u}{g}-n\right)^2$$

$$\Rightarrow u^2=2v\,(u-ng)-(u-ng)^2$$

$$\Rightarrow -2u^2+2uv-2nvg+2nug=n^2g^2$$

$$\Rightarrow 2u\,(v-u)-2ng\,(v-u)=n^2g^2$$

$$\Rightarrow 2\,(v-u)\,(u-ng)=n^2g^2$$

$$\Rightarrow v-u=\frac{n^2g^2}{2\,(u-ng)}.$$

Ex. 7 A stone is dropped into a well and the sound of splash is heard after 4 seconds Assuming the velocity of sound to be 340 m/s, determine the depth of the well.

Let h m be the depth of the well and let t seconds be the time taken by the stone to reach the surface of water. Since the total time taken by the stone and the sound is 4 seconds, the sound takes $(4-t)$ seconds to describe a distance of h.

Considering the vertically downwand motion of the stone and using the formula $h=ut+\frac{1}{2}gt^2$, we get

$$h=0+\frac{1}{2}\times 9.8\,t^2$$

$$=4.9\,t^2 \qquad ..(1)$$

Now, considering the uniform motion of the sound, we get

$$h=340\times(4-t)=1360-340\,t \qquad ...(2)$$

From (1) and (2), we get

$$4.9t^2+340t-1360=0.$$

$$\Rightarrow \quad t=\frac{-340\pm\sqrt{340^2+4\times 4.9\times 1360}}{2\times 4.9}$$

$$=\frac{-340\pm\sqrt{115600+26666}}{9.8}$$

$$=\frac{-340\pm\sqrt{142256}}{9.8}=\frac{-340\pm 377.17}{9.8}.$$

Since t can not be negative, we take the positive value of t. Thus $t=\frac{37.17}{9.8}=3.79.$

$\therefore$ From (1),

$$h=4.9\times 3.79^2$$

$$=70.38.$$

Hence, the required depth of the well is 70.38 m.

Ex. 8. A man in a lift, ascending, with an acceleration f m/s^2 throws a ball vertically upwards with a velocity v m/s relative to the lift, and catches it again it t seconds. Show that $f+g=\frac{2v}{t}$. [C. U. 44]

The lift is moving upwards with an acceleration f m/s^2 and the ball moves vertically downwards, freely under gravity with acceleration g, when it is released from the lift. Hence, the acceleration of the ball relative to the lift$=(f+g)$ m/s^2 downwards The displacement of the ball relative to the lift in time t is zero. Hence, considering the upward motion of the ball and using the formula

$$h=ut-\frac{1}{2}at^2, \text{ we get}$$

$$0=vt-\frac{1}{2}(f+g)\,t^2$$

$$\Rightarrow \quad f+g=\frac{2v}{t}.$$

Ex. 9. From an aeroplane rising vertically with uniform acceleration f, a ball is dropped ; 4 seconds after this, another ball is dropped from the aeroplane. Show that the distance between the two balls 2 seconds after the second ball is dropped is $16(g+f)$. Let A be the position of the aeroplane when the first ball is dropped During 4 seconds, let the position of the aeroplane be at B. Let u and v be the velocities of the aeroplane at A and B respectively The first ball is dropped from A and the second from B Let Q be the position of the second ball 2 seconds after it is dropped from B and let P be the position of the first ball.

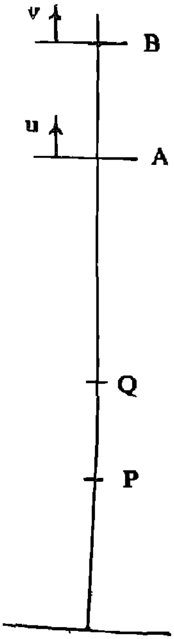


Fig. 42

Now, considering the motion of the aeroplane from A to B and using the formula $v=u+at$, we get

$$v=u+4f \qquad \ldots(1)$$

It should be noted that the aeroplane is moving up against gravity with its own acceleration f and hence we have used $v=u+4f$ and not $v=u-4f$. Had a body been projected up with initial velocity u, it would move up freely under gravity with retardation g and in that case we would use the formula

$$v=u-gt.$$

Also, the distance AB has been traversed by the aeroplane in 4 seconds.

Hence, $AB=4u+\frac{1}{2}f.4^2=4u+8f \qquad \ldots(2)$

Now, considering the motion of the second ball from B to Q and taking the downward direction as positive, we have, by using the formula $h=ut+\frac{1}{2}gt^2$,

$$BQ=-v.2+\frac{1}{2}g.2^2$$

$$=-2(u+4f)+2g \qquad \dots(3)$$

[from (1)]

In the above, the negative sign of the initial velocity of the second ball has been taken. This is because the aeroplane is moving up with velocity v at B and hence the initial velocity of the ball when it is dropped from the aeroplane is also v in the upward direction. The initial velocity of this ball is, therefore, $-v$ in the downward direction which has been chosen as the positive direction. Since the first ball had been dropped 4 seconds earlier than the second ball, the first ball will take (4+2) seconds or 6 seconds to travel the distance AP. Now, considering the motion of the first ball from A to P and using the formula $h=ut+\frac{1}{2}gt^2$, we get

$$AP=-6u+\frac{1}{2}g.6^2=-6u+18g \qquad \dots(4)$$

Hence, the required distance between the two balls two seconds after the second ball is dropped

$$=PQ=BP-BQ$$

$$=BA+AP-BQ$$

$$=4u+8f-6u+18g+2(u+4f)-2g$$

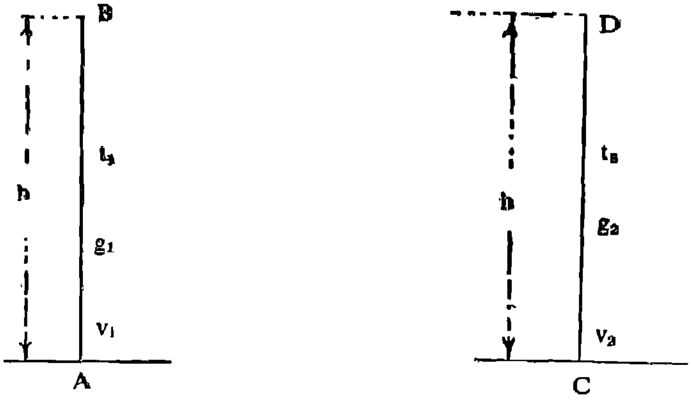
[from (2), (3), and (4)]

$$=16\,[g+f].$$

Ex. 10. If a particle takes n seconds less and acquires a velocity of p m/s more at one place where acceleration due to gravity is g_1 than at another where acceleration due to gravity is g_2, on falling through the same distance, show that

$$g_1g_2=\frac{p^2}{n^2}.$$

Let A and C be the two places where the accelerations due to gravity are g_1 and g_2 respectively. Let the particle be dropped at A from B and the same particle be dropped at C from D, where $AB=CD=h$. Let t_1 and t_2 be the times taken by the particle to travel distances BA and DC



(*i*) (*ii*)

Fig. 43

respectively and v_1, v_2 be the final velocities of the particle at A and C respectively. It is given that

$$t_2-t_1=n \qquad \text{..(1)}$$

and $$v_1-v_3=p \qquad \text{..(2)}$$

Considering the motion of the particle from B to A and using the formula $h=ut+\frac{1}{2}gt^2$ and $v^2=u^2+2gh$

$$h=\frac{1}{2}g_1t_1^2 \qquad \text{..(3)}$$

$$v_1^2=2g_1h \qquad \text{...(4)}$$

Similarly, considering the motion of the particle from D to C, we get

$$h=\frac{1}{2}g_2t_2^2 \qquad \text{..(5)}$$

and $$v_2^2=2g_2h \qquad \text{...(6)}$$

From (3) and (6), we get

$$t_2-t_1=\sqrt{\frac{2h}{g_3}}-\sqrt{\frac{2h}{g_1}}=\frac{\sqrt{2g_1h}-\sqrt{2g_2h}}{\sqrt{g_1g_2}}$$

Hence, using (4) and (5), we get

$$t_2-t_1=\frac{v_1-v_2}{\sqrt{g_1g_2}}$$

$$\Rightarrow \quad n=\frac{p}{\sqrt{g_1g_2}} \qquad \text{[from (1) and (2)]}$$

$$\Rightarrow \quad g_1g_2=\frac{p^2}{n^2}.$$

Ex. 11. A particle starts from rest from the top of a smooth inclined plane of a given base. Show that the time of fall is least when the inclination of the plane to the horizon is 45°. [C.U. 38]

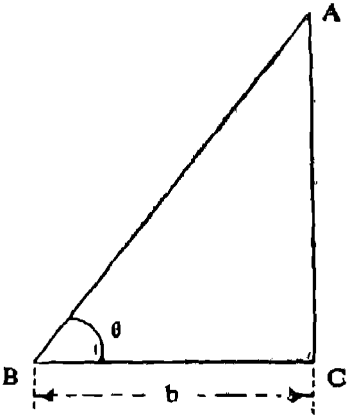


Fig. 44

Let AB be an inclined plane with base BC and height AC. Let $BC=b$, $\angle ABC=\theta$ and t be the time from A to B. Considering the motion down the plane and using the formula $h=ut+\frac{1}{2} g \sin \alpha .\, t^2$, we get

$$AB=\frac{1}{2} g \sin \theta . t^2 \qquad \ldots(1)$$

But from $\triangle ABC$, we have

$$\cos \theta=\frac{b}{AB} \Rightarrow AB=\frac{b}{\cos \theta} \qquad \ldots(2)$$

Hence, from (1) and (2),

$$\frac{b}{\cos \theta}=\frac{1}{2} g \sin \theta .\, t^2$$

$$\Rightarrow \quad t^2=\frac{2b}{g} \cdot \frac{1}{\sin \theta \cos \theta}=\frac{4b}{g} \; \frac{1}{\sin 2\theta}$$

$$\Rightarrow \quad t=2 \sqrt{\frac{b}{g}} \cdot \frac{1}{\sqrt{\sin 2 \theta}}.$$

Now, since b is given, t is least when $\sin 2\theta$ is greatest i.e. when $2\theta=90^\circ=\theta\Rightarrow 45^\circ$.

Hence the result.

Ex. 12. A particle is allowed to slide down a smooth inclined plane from its top and after describing $\frac{3}{4}$ th of the length, it passes a second particle which was simultaneously projected up the plane from the foot.

What fraction of the total height of the plane does the second particle rise ?

Let AB be the inclined plane of length with base BC and height $AC=h$. Let $\angle ABC=\theta$.

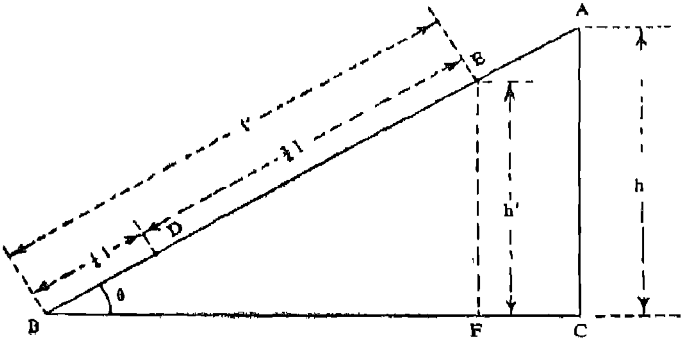


Fig. 45

Let the two particles meet at D after time t, where $AD=\frac{3}{4}.l$ and $BD=\frac{1}{4}\ l$. Let u be the initial velocity of the second particle which is thrown up the plane from B. Now, considering the upward motion of the second particle from B to D and using the formula $h=ut-\frac{1}{2}\ g \sin\alpha\ .\ t^2$, we get

$$\frac{1}{4}\ l=ut-\frac{1}{2}\ g\sin\theta\ .\ t^2 \qquad \text{...(1)}$$

Again, considering the downward motion of the first particle from A to D, we get

$$\frac{3}{4}\cdot l=\frac{1}{2}\ g\sin\theta\ .\ t^2 \qquad \text{...(2)}$$

Adding (1) and (2), we get

$$l=ut \quad \Rightarrow \quad t=\frac{l}{u}.$$

Substituting the value of t in (2), we get

$$\frac{3}{4}\ l=\frac{1}{2}\ g\sin\theta\ .\ \frac{l^2}{u^2}$$

$$\Rightarrow \quad l=\frac{3}{2}\cdot\frac{u^2}{g\sin\theta}\ . \qquad \text{...(3)}$$

Let the second particle ascend up the plane upto the point E where its velocity becomes zero. Let $BE=l'$ and the height of E above the base BC i.e. $EF=h'$ Considering the upward motion of the second particle from B to E and using the formula $v^2=u^2-2\,g\sin\alpha\,.\,h$, we get

$$0=u^2-2gl'\sin\theta$$

$$\Rightarrow\quad l'=\frac{u^2}{2g\sin\theta} \qquad \ldots(4)$$

Hence, from (3) and (4), we get

$$\frac{l'}{l}=\frac{u^2}{2g\sin\theta}\times\frac{2g\sin\theta}{3u^2}=\frac{1}{3}\,.$$

Ex. 13. A particle sliding down a smooth inclined plane is observed to pass over two consecutive equal distances of 3 cm in $\frac{1}{2}$ s and $\frac{1}{4}$ s respectively. Find the inclination of the plane to the horizon. (Take $g=10\text{m/s}^2$)

Let P, Q and R be three consecutive points on the inclined plane AB such that $PQ=QR=3$ cm$=.03$ m. Let $\angle ABC=\theta$, where BC is the base and AC is the height of the inclined plane. Let u be the velocity at P. The distance $PQ=3$ m is described by the particle in $\frac{1}{2}$ s and the distance PR is described by the particle in $\left(\frac{1}{2}+\frac{1}{4}\right)$ s$=\frac{3}{4}$ s.

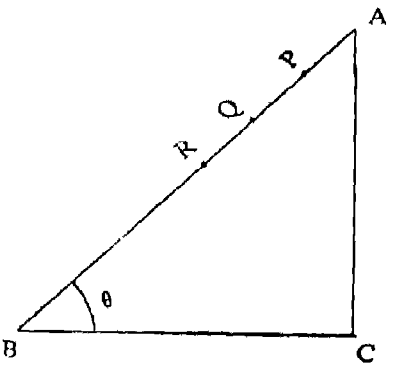


Fig. 46

Hence, considering the motion of the particle down the plane from P to Q and from P to R successively and using the formula $h=ut+\frac{1}{2}\,g\sin\alpha\,.\,t^2$, we get

$$.03=u.\frac{1}{2}+\frac{1}{2}\times10\times\sin\theta.\left(\frac{1}{2}\right)^2$$

$$\Rightarrow\quad .12=2u+5\sin\theta \qquad \ldots(1)$$

$$\text{and}\quad .06=u\;\frac{3}{4}+\frac{1}{2}\times\;10\times\sin\theta.\left(\frac{3}{4}\right)^2$$

$$\Rightarrow\quad .96=12u+45\sin\theta$$

$$\Rightarrow\quad .32=4u+15\sin\theta \qquad \ldots(2)$$

Multiplying (1) by 2 and subtracting from (2), we get

$$.08=5\sin\theta$$

$$\Rightarrow \quad \sin\theta=\frac{.08}{5}=\frac{8}{500}=\frac{2}{125}.$$

Hence, $\theta=\sin^{-1}\left(\frac{2}{125}\right)$ giving the inclination of the plane with the horizon.

Ex. 14. *AB* is a quadrant of a circle with centre *O*, *OA* being horizontal. *C* is a point on the quadrant such that $\angle AOC=2\theta$. Prove that the interval of falling from *A* to *C* and that of falling from *C* to *B* are in the ratio $\sqrt{\sin\theta}\ .\ \sqrt{\cos\theta}$.

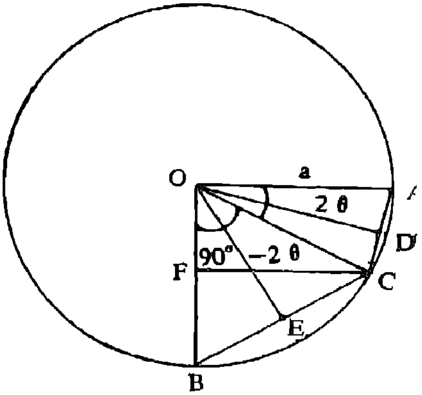


Fig. 47

We join *AC* and *CB*. Let $OD\perp AC$ and $OE\perp BC$ Let *a* be the radius of the circle.

Since $\angle AOD=\frac{1}{2}\angle AOC=\theta$,

$$\therefore \quad AD=\sin\theta$$

$$\therefore \quad AC=2\ AD=2a\sin\theta.$$

$$\angle COB=90^\circ-2\theta.$$

$$\therefore \quad \angle COE=\frac{1}{2}\angle COB$$

$$=45^\circ-\theta$$

$$\therefore \quad CE=a\sin(45^\circ-\theta)$$

$$\therefore \quad CB=2CE=2a\sin(45^\circ-\theta).$$

Now, $\angle OAC=90^\circ-\theta$.

If *CF* is drawn paralled to the horizontal *OA*, then

$$\angle BCF=90^\circ-\angle OBC$$

$$=90^\circ-(45^\circ+\theta)=45^\circ-\theta.$$

Let t_1 and t_2 be the times from *A* to *C* and that from *C* to *B* respectively. The lines *AC* and *CB* will make angles θ and 45°+θ respectively with the vertical. Hence, the component of the vertically downward acceleration *g* along *AC* is $g\cos\theta$ and that along *CB* is $g\cos(45^\circ+\theta)$. Now, considering the motion of the point from *A* to *C* and from *C* to *B*, we get

$$AC=\frac{1}{2}g\cos\theta\ .\ t_1^2$$

$$\Rightarrow \quad 2a \sin \theta = \frac{1}{2} g \cos \theta . t_1^2$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{4a \sin \theta}{g \cos \theta}}$$

and $\quad CB = \frac{1}{2} g \cos (45^\circ + \theta) . t_2^2$

$$\Rightarrow \quad 2a \sin (45^\circ - \theta) = \frac{1}{2} g \cos (45^\circ + \theta) . t_2^2$$

$$\Rightarrow \quad t_2 = \sqrt{\frac{4a \sin (45^\circ - \theta)}{g \cos (45^\circ + \theta)}} = \sqrt{\frac{4a \sin (45^\circ - \theta)}{g \sin (45^\circ - \theta)}} = \sqrt{\frac{4a}{g}} .$$

Hence, $\quad \frac{t_1}{t_2} = \sqrt{\frac{4a}{g}} \sqrt{\frac{\sin \theta}{\cos \theta}} : \sqrt{\frac{4a}{g}}$

$$= \sqrt{\sin \theta} : \sqrt{\cos \theta} .$$

Ex. 15. A parabola has its axis vertical and its vertex at the lowest point. Prove that the time of descent of a particle down any smooth chord to the lowest point is equal to that of falling vertically to the horizontal line which is at a depth below the vertex equal to the latus rectum.

Let AP be any chord passing through the vertex A of the parabola, P being any point on the parabola. Let AY be the axis of the parabola and we draw $PN \perp AY$.

Let $\angle PAN = \theta$. We take the tangent at the vertex as the x-axis and the axis of the parabola as the y-axis. Let P be the point (x, y). Let the equation of the parabola be $x^2 = 4ay$, where $4a$ is the length of the latus rectum and $(0, a)$ are the coordinates of the focus S of the parabola. We draw a straight line MB parallel to x-axis and at a distance $4a$ from the vertex away from the parabola.

Let t be the time of descent down the chord PA and T, the time of fall vertically from P to the line MB.

Considering the motion of the particle from P to A, we get

$$PA = \frac{1}{2} g \cos \theta . t^2 \qquad ..(1)$$

Considering the vertical motion of the particle from P to M, we get

$$PM = \frac{1}{2} g T^2 \qquad ...(2)$$

From (1),

$$PA = \frac{1}{2} g t^2 . \frac{AN}{AP}$$

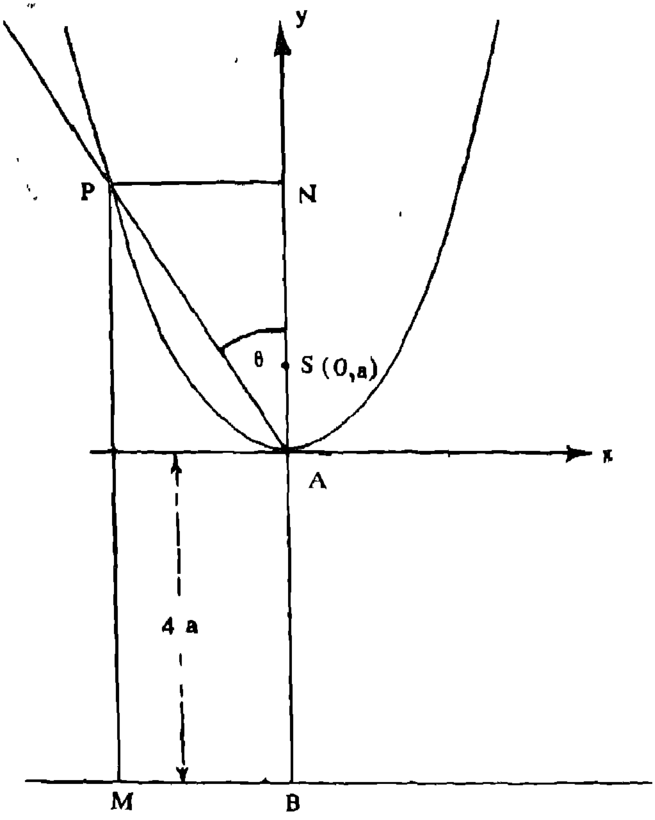


Fig. 48

$$\Rightarrow \quad AP^2 = AN.\frac{1}{2}\,gt^2$$

$$\Rightarrow \quad x^2 + y^2 = y\;\frac{1}{2}gt^2$$

$$\Rightarrow \quad 4ay + y^2 = y\;\frac{1}{2}gt^2$$

[From the equation of the parabola $x^2 = 4ay$]

$$\Rightarrow \quad y + 4a = \frac{1}{2}gt^2 \qquad \ldots(3)$$

From (2),

$$AN+AB=\frac{1}{2}gT^2$$

$$\Rightarrow \quad y+4a=\frac{1}{2}gT^2 \qquad \ldots(4)$$

From (3) and (4), we get

$$t=T.$$

Hence the result.

EXERCISE 5

Note : Take $g=9.8\ m/s^2$ unless otherwise stated.

1. What is the height of a river bank if a stone falling from it reaches the water in 3 seconds? What is the final velocity of the stone? [Ans. 44.1 m, 29.4 m/s]
2. Two loads are dropped from a helicopter with zero initial velocity is after each other. How far apart are the loads 2s and 4s after the first load was dropped ? [Ans 14.7m and 34.3m]
3. Compare the time of free fall from equal altitude above the moon's and the earth's surfaces, if the acceleration of falling bodies on the moon is $1.6 m/s^2$. [Ans. $2\sqrt{2}:7$]
4. From the top of a tower 98m high a body is projected vertically upwards with a velocity of 39.2m/s. Find
   (i) the greatest height attained from the top of the tower
   (ii) the velocity with which it strikes the ground and
   (iii) the time during which it remains in the air.
   [Ans. (i) 78.4m (ii) 58.8m/s (iii) 10s]
5. A ball is projected vertically upwards with a velocity of 19.6m/s. Find
   (i) the time taken to reach the greatest height.
   (ii) the greatest height.
   (iii) the position of the ball after $t=5$ seconds
   (iv) the time taken to reach a height of 14.7m.
   [Ans. (i) 2s (ii) 19.6m
   (iii) 24.5m below the point of projection (iv) 1s and 3s]
6. A stone is projected vertically upwards with a velocity of 98m/s. Find

(i) the position of the stone when $t=20$s.

(ii) how far the stone travels during the 11th second of motion.

(iii) the time at which the stone is at a height of 93.1m.

[Ans. (i) At the point of projection. (ii) 4.9m ; this is the distance fallen in 1s from the greatest height (iii) 1s and 19s]

7. A man is throwing four balls simultaneously in such a way that while he is throwing one ball, other three are then in air. If each ball attains a maximum height of 4 9m

   (i) find the velocity of projection of each ball.

   (ii) what are the positions of three balls with respect to thrower's hand when he throw's a particular ball ?

   [Ans (i) 9 8m/s (ii) 1 225m, 4 9m and 1.225m above his hand]

8. The ram of a pile-driver is lifted 4.9m is 5s It then falls on the pile How many blows can the pile-driver make per minute ? [Ans. 10]

9 A hammer falls freely from a height of 1.25m. What is its velocity just before striking ? (Take $g=10\text{m/s}^2$) [Ans 5m/s]

10. A falcon dives for its prey with a velocity of 100m/s. From what height must it dive to attain this speed ? (Consider the flight of the falcon as a free fall) (Take $g=10\text{m/s}^2$) [Ans 500m]

11. From what height did a body fall if it dropped 75m during the last second of the fall ? (Take $g=10\text{m/s}^2$) [Ans. 320m]

12. A small but heavy body was dropped trom a balloon at an altitude of 1125 m How long did it take to reach the ground if

    (i) the balloon was motionless ?

    (ii) it was descending at a velocity of 15 cm/s ? (The initial speed of the body relative to the balloon was zero)

    (Take $g=10\text{ m/s}^2$)

    [Ans (i) 15 s (ii) 13.6 s]

13. A body falling from rest under gravity passes a certain point with a velocity of 120 m/s From what height above the point did the body begin to fall ? Where was it 2s previously and where will it be 2s later ?

    (Take $g=10$ m/s)

    [Ans. 720 m; 500 m; 980 m]

14. A body is projected upwards with a velocity of 14.7 m/s. After what time will it be respectively at heights of 9.8 m, 11.025 m and 15 m above and 19.6 m below the point of projection ?

[Ans. 1s or 2s; $1\frac{1}{2}$ s; cannot rise so high; 4s]

15. A stone is projected vertically upwards with a velocity sufficient to carry it to a height of 50 m. Find its velocity when it is halfway up. [Ans. $7\sqrt{10}$ m/s]

16. A body falls freely from rest at the top of a tower 98m high. Calculate the time taken by the body to reach the ground and the velocity of impact. [Ans. 4.5s, 43.8m/s.]

17. It is safe to jump from the height of 2m on the surface of the earth. What is the corresponding safe height on the surface of a planet where the gravitational acceleration is 1.96m/s^2 ? (Assume the surfaces of both the planets to be equally hard.)

[Ans. 10m]

18. A jet plane is rising vertically with a velocity of 10m/s. It has reached a certain height when the pilot drops a coin, which takes 4 seconds to hit the ground. Assuming that there is no resistance to the motion of the coin, calculate the height of the plane and the velocity of the coin on impact with the ground.

[Ans. 38.4m, 29.2m/s]

19. *P* and *Q* are two points in the same vertical line, *P* being above *Q*. A heavy particle is projected vertically upwards from *Q* with velocity which will just carry it to *P* and at the same time a heavy particle is dropped from rest at *P*. Show that when the particles meet, their velocities will be equal and opposite and the space passed over by the particles will be as 3 : 1.

20. A stone is dropped down a well and hits the water after 3s. Calculate the depth of the well and the velocity of the impact.

[Ans. 44.1m ; 29.4 m/s]

21. A brick is dropped into a well in which the level of water is 30m below the top of the well. What is the time after which the sound of the splash is heard ? Velocity of sound in air is 332m/s

[I.I.T.61; AIIMS 67]

[Ans. 2.56s]

22. A stone is dropped into a well in which the level of water is 78.4 m below the top of the well. The sound of the splash is heard 4.24 s later. What is the speed of sound in air ?

[Ans. 326.6 m/s]

23. A cricket ball of mass 150 g is thrown vertically upwards with an initial velocity of 29.4 m/s. Calculate (i) the time taken by the ball to return to the thrower (ii) the velocity with which it returns to the thrower and (iii) the maximum height attained by the ball. [AIIMS 69]
[Ans. (i) 6s (ii) 29.4 m/s downwards (iii) 44 1 m]

24. A body is projected vertically upwards from the top of a tower with a velocity of 29.4 m/s. If the height of the tower be 78.4 m, calculate the time taken by the body to reach the ground below [A.I.D.M.S 75]
[Ans. 8s]

25 A boy throws a ball vertically upwards from the ground to the top of the school flag-pole. It returns to the ground after 2s What is the height of the pole ? [AIIMS 73]
[Ans. 4.9 m]

26. A ball is thrown vertically upwaros is caught by the thrower after 9s.
(i) How high did the ball rise ?
(ii) With what velocity was it thrown ?
(iii) What was its heights 6s after projection ?
[Ans. (i) 99.225 m, (ii) 44.1 m, (iii) 88 2 m]

27. An anti-aircraft shell is fired vertically upwards with a velocity of 490 m/s.
(i) What is the maximum height attained ?
(ii) What time does it take to reach this height ?
(iii) When will its height be 4410 m ?
[Ans. (i) 12.25 km, (ii) 50s, (iii) 10s and 90s]

28 A balloon is ascending at the rate of 14 m/s at a height of 98 m above the ground when a packet is dropped from the balloon. After how much time and with what velocity does it reach the ground ? [Ans. 6.12 s, 46.02 m/s in the downward direction]

29. A ball is thrown vertically upwards with a speed of 19.6 m/s from the top of a tower and returns to the earth in 6s. Find the height of the tower. [A.I.S.S.C. 78]
[Ans. 588 m]

30. A foot packet is released from a helicopter which is rising steadily at 2 m/s. After 2s (i) what is the velocity of the packet ? (ii) How far is it below the helicopter ? [D.B. S.S.C. 84]
[Ans. (i) 17.6 m/s in the downward direction
(ii) 15.6 m below the helicopter]

31. A bag released from a balloon at rest falls freely for 10s before hitting an over-head transmission wire. The bag loses $\frac{6}{7}$th of its velocity due to impact of the wire and then strikes the ground below after 1s. What was the height of the wire ?

[Ans. 18.9 m]

32. A parachutist bails out from an aeroplane and after dropping through a distance of 40 m, he opens the parachute and decelerates at 2 m/s^2. If he reaches the ground with a speed of 2 m/s, how long is he in the air ? At what height did he bail out from the plane ? [Ans. $15\frac{6}{7}$ s ; 235 m]

33. A body is dropped from rest at the height of 150 m and simultaneously another body is dropped from rest from a point 100 m above the ground. What is the difference in heights after they have fallen (i) 2s (ii) 3s ? [Ans. (i) 50 m (b) 50 m]

34. A ball is allowed to fall from a tower 2000 m high. As soon as the ball reaches half-way, the acceleration due to gravity suddenly vanishes. Discuss the situation.

[Ans The ball will fall with a constant velocity of 140 m/s]

35. From an elevated point A, a stone is projected vertically upwards, when the stone reaches a distance h below A, its velocity is double of what it was at a height h above A. Show that the greatest height attained by the stone above A is $\frac{5h}{3}$.

[A.I.I.M.S. 73]

36. From a point P, 10 m above the ground, a stone is projected vertically upwards, when it is 6 m below P, its velocity is double of what it was at a height of 6 m above P Show that the point P is exactly midway between the ground and the highest point.

37. A bag released from a balloon at rest covers 24.5 m during the last second of its motion. (i) What is the height of the balloon ? (ii) With what velocity does the bag hit the ground ?

[Ans. (i) 44.1 m (ii) 29.4 m/s]

38. Two bodies are thrown vertically upwards with their velocities in the ratio 2 : 5. What will be the ratio of the maximum heights attained by them ? [Ans. 4 : 25]

39. In a circus show, a juggler throws a ball with a velocity of 14.7 m/s vertically upwards. At the same instant, another ball starts falling along a parallel line from rest from a height of 9.8 m. (i) What will be their velocities at the instant of crossing each other? (ii) At what height do they cross each other?

[Ans. (i) 8.16 m/s upwards, 6.53 m/s downwards (ii) 7.62 m]

40. Two balls are projected vertically upwards at the same instant along parallel lines from the ground. One ascends 34.3 m higher than the other and returns to the ground 2s later. Find the velocities of projection of the balls and the heights attained by them. [Ans. 39.2 m/s ; 29.4 m/s ; 78.4 m ; 44.1 m]

41. A bag is dropped from a balloon ascending at the rate of 12 m/s vertically, when it is 80 m above the ground. How much time does the bag take to reach the ground? [Ans. 5.4 s]

42. A food bag is dropped from a helicopter, ascending with a uniform vertical velocity, and reaches the ground below in 10s. When the bag reaches the ground, the helicopter was at a height of 490 m. With what velocity was the helicopter ascending?

[Ans. 5 m/s]

43. A parachutist drops freely from an aeroplane for 10s before the parachute opens out. Then he descends with a net retardation of 2 m/s^2. Find the height at which he gets out of the aeroplane, if he reaches the ground with a velocity of 8 m/s. [I.I.T 62]

[Ans. 2875 m]

44. A cage in a mine shaft is rising with a velocity of 15 m/s. A piece of coal which falls from the cage takes 8 s to reach the bottom of the shaft. Find how high the case was above the base of the shaft when the piece of coal fell out, and also the velocity of impact. [Ans. 193.6, m ; 63.4 m/s]

45. A stone is thrown vertically downwards with an initial velocity of 40 m/s. If it takes 2s to reach the ground, from what height was the stone projected? Find also the velocity of impact.

[Ans. 99.6 m ; 59.6 m/s]

46. A ball falls from a height of 9.8 m and rebounds to a height of 4.9 m. Calculate the time from the moment of release to the second impact with the ground. [Ans. $(\sqrt{2}+2)$ s]

How many times must the initial velocity of a ball thrown up vertically be increased in order to double (i) its maximum height and (ii) the time of its upward motion ?

[Ans. (i) $\sqrt{2}$ times (ii) 2 times]

A fountain must rise not less than 2.5 m and not more than 5 m. What initial velocity must the jet have ?

[Ans. If V_0 m/s be the initial velocity, then $7.0 < V_0 < 7\sqrt{2}$]

A balloon starts rising with aa acceleration of 2 m/s^2. 5 seconds afterwards, ballast was released with a zero initial velocity relative to the balloon. How long will it take the ballast to reach the ground ? Assume that $g = 10$ m/s^2. [Ans $(1+\sqrt{6})$ s].

A body moving in a vertically upward direction passes a point at a height of 54.5 cm with a velocity of 436 cm per second ; with what initial velocity was it thrown up and how long further will it rise ? [Ans. 545 cm/s ; $\frac{4}{9}$ s]

A body is projected vertically upwards with a certain velocity and it is found that when it is 500 m from the ground, it takes 10 s to return to the same height again. Find the velocity of projection and the whole time of flight

[Ans. 110.46 m/s ; 22.5 s]

A block falls from a mast-head and is observed to take $\frac{2}{5}$ s in falling from deck to the bottom of the hole, a distance of 9.8 m. Calculate the height of mast-head above the deck.

[Ans. 25.947 m]

Two bodies are projected from the top of a building, one vertically upwards with a velocity of 35 m/s and the second vertically downwards with an initial velocity of zero The second body starts 5 seconds after the start of the first. It is found that both the bodies strike the ground at the same time. Determine the height of the building. [Ans. 68.91 m]

A body is projected vertically upwards with an initital velocity of 24.525 m/s. Find the height to which the body rises. Another body is projected vertically upwards 2s after the first with an initial velocity of 19.62 m/s. Find when the two bodies are at the same height and find also the velocity of each body at this instant. [Ans 30 69 m ; 2 s after the projection of second ; 14.675 m/s downwards and zero]

55. A stone falling from the top of a vertical tower has descended x m when another is let fall from a point y m below the top. If they fall from rest and reach the ground together, show that the height of the tower is $\frac{(x+y)^2}{4x}$ m [C.U. 36]

56. A stone falls freely for 3 s when it passes through a sheet of glass and loses half its velocity and then reaches the ground in $\frac{1}{2}$ s Find the height of the glass above the ground. [Ans. 15.925 m]

57 A, B, C, D are points in a vertical line, the length AB, BC, CD being equal If a body falls freely from A, prove that the times of describing AB, BC, CD are respectively as

$$1 . (\sqrt{2}-1) \quad (\sqrt{3}-\sqrt{2})$$

58. A stone dropped into an empty pit of depth h is heard to strike the bottom after t s. Prove that

$$2h\left(1+\frac{gt}{v}\right)=gt^2,$$

where v is the velocity of sound supposed so large compared with h that $\left(\frac{h}{v}\right)^2$ can be neglected. [C.U. 33]

59. A body projected vertically upwards from the top of a tower reaches the ground in t_1 seconds. If projected vertically downwards with the same velocity, it reaches it in t_2 seconds Prove that if simply let drop, it would reach the ground in $\sqrt{t_1 t_2}$ s.

60. A stone is dropped from a certain height and is observed to fall the last h m in t s. Show that the total time of fall is

$$\left(\frac{t}{2}+\frac{h}{gt}\right) \text{ s.}$$

61. Bodies are simultaneously projected vertically downwards from heights h_1, h_2, h_3 with velocities v_1, v_2, v_3 respectively, and they all reach the ground at the same moment. Show that

$$\frac{h_1-h_2}{v_1-v_2}=\frac{h_2-h_3}{v_2-v_3}=\frac{h_3-h_1}{v_3-v_1}.$$

If instead of projecting the two bodies downwards, if they are thrown vertically upwards from heights h_1, h_2, h_3, prove that

$$v_1(h_2-h_3)+v_2(h_3-h_1)+v_3(h_1-h_2)=0.$$

62. A particle is projected in the vertically upward direction. The velocities of the particles at heights x_1, x_2 and x_3 from the point of projection are respectively v_1, v_2 and v_3. Show that the acceleration g due to gravity will be

$$g=\frac{v_1^2x_1(x_2-x_3)+v_2^2x_2(x_3-x_1)+v_3^2x_3(x_1-x_2)}{2(x_2-x_3)(x_3-x_1)(x_1-x_2)}.$$

63. A boy in a lift, which is descending with uniform acceleration f, drops a marble at a height h from the floor of the lift at an instant when the velocity of the lift is v. How long will the marble take to reach the floor ? Test your result by putting $f=0$ and $f=g$ in turn. What will be the answer if the lift is rising instead of descending ? $\left[\text{Ans. } \sqrt{\frac{2h}{g-f}}\ ;\ \sqrt{\frac{2h}{g+f}}\right]$

64 A lift starts from the top of a mine shaft and descending with a constant speed of 10 m/s ; 4 seconds later a boy throws a stone vertically upwards from the top of the shaft with speed 30 m/s. Find when and where the stone hits the lift. (Take $g=10$ m/s^2)

[Ans. After 12.90 s after the start of the lift ;
129.0 m below the top of the shaft.]

65. A particle is projected vertically upwards with a velocity of u m/s and after t s, another particle is projected upwards from the same point with the same initial velocity. Prove that they will meet at a height of

$$\frac{4u^2-g^2t^2}{8g} \text{ m after } \left(\frac{t}{2}+\frac{u}{g}\right) \text{ seconds}$$

66 A rocket ascending vertically from the ground with an initial velocity of $\sqrt{2gy}$ m per second explodes when it reaches the greatest height, and the interval between the sound reaching the place of starting and a place distant x m from it is $\frac{1}{n}$th of a second. Show that the velocity of sound is $n(\sqrt{x^2+y^2}-y)$ m/s.

67 The depth of a well is to be measured by dropping in a stone and observing the time T which then elapses before the impact

at the bottom is heard. Prove that the depth of the well is given by the smaller root of the equation

$$gx^2-2x(v^2+gVT)+gV^2T^2=0,$$

where V is the assumed constant speed of sound in air.

Hence, show, with the help of a binomial expansion, that if gT is small compared with V, the depth of the well is approximately

$$\frac{1}{2}gT^2-\frac{1}{2}g^2\frac{T^3}{V}.$$

68 With what velocity must a particle be projected up a plane 10 m in length and inclined to the horizon at an angle of 30° so as to reach the top in one second ? [Ans. 12.45 m/s]

69. A particle is projected (i) upwards, (ii) downwards on a plane inclined to the horizon at an angle of 30°. If the initial velocity be 8 m/s in each case, find the distances described and the velocities acquired in 4 s.

[Ans. (i) 7.24 m below the point from where it started ; 11 62 m/s (ii) 71 24 m down the plane ; 27 62 m/s]

70. A heavy particle slides from rest down a smooth inclined plane which is 15 m long and 12 m high. What is its velocity when it reaches the ground, and how long does it take ?

[Ans 15.342 m/s ; 1.95 s]

71. A particle sliding down a smooth plane 1962 cm long acquired a velocity of 981 cm/s starting from rest Find the inclination of the plane. $\left[\text{Ans. } \sin^{-1}\frac{1}{4}\right]$

72. A body is projected with a velocity of 1660 cm/s up a smooth inclined plane of inclination of 30°. Find the distance described up the plane, and the time that elapses before it comes to rest.

[Ans. 2809 cm ; 3.39 s]

73. An engine rises up an incline of 1 in 20 with a uniform acceleration of 2 m/s^2 starting from rest at the foot. After a certain distance the steam is shut off and the impetus just carries the engine to the top. If the length of the incline be 5 km, find where the steam was shut off. (Take $g=10$ m/s^2)

[Ans. 1 km from the bottom]

74. A smooth inclined plane of length l and height h is fixed on a horizontal plane. Find the velocity with which a particle must be projected down the plane from the top in order that it may reach the horizontal plane in the same time as a particle let fall vertically from the top. $\left[\text{Ans. } \dfrac{l^2-h^2}{l}\sqrt{\dfrac{g}{2h}}\right]$

75. A particle slides without friction down an inclined plane and in the 5th second after starting passes over a distance of 2207.25 cm ; find the inclination of the plane to the horizon. [Ans. 30°]

76. Two smooth inclined planes of the same altitude and of elevation α and β stand back to back. A body projected up the first plane from its foot along the line of greater slope with a velocity u, ascends it, and without losing any velocity at the turn, descends the second plane. Find its velocity at the foot of the second plane. [Ans. u]

77. Two heavy particles begin to slide at the same instant from the common vertex of two smooth inclined planes. Prove that the line joining them moves, remaining always parallel to itself.

78. A body moving down a smooth inclined plane is observed to fall through equal spaces s in consecutive intervals t_1, t_2, prove that the sine of the inclination of the plane to the horizon is

$$\frac{2s\,(t_1-t_2)}{gt_1\,t_2\,(t_1+t_2)}$$

79. Two particles slide down two straight lines in the same vertical plane, at right angles to one another, starting simultaneously from rest from their point of intersection. Prove that their distance apart at any time will be equal to the distance either of the two particles would have descended vertically in that time

80. If a chord is drawn from one end of the horizontal diameter to any point of a vertical circle, show that the time the particle would take in sliding down that chord would vary as the square root of the tangent of the inclination of the chord to the vertical. [U.P. 38]

81. In a vertical circle, two chords are drawn from an extremity of a horizontal diameter, subtending angles α and 2 α at the centre. If the times of sliding down these chords be as 1 : n, Show that sec $\alpha=n^2-1$.

82. *AB* is a vertical diameter of a circle whose plane is vertical and *PQ*, a diameter inclined at an angle θ to *AB* Find θ so that the time of sliding down *PQ* may be twice that of sliding down *AB*. $\left[\text{Ans. } \cos^{-1}\frac{1}{4}\right]$

83. A particle slides down a smooth chord of a vertical circle ending in the lowest point Show that the velocity acquired varies as the length of the chord.

84. *APB* and *AQC* aee two circles with their centres in the same vertical line *ABC*, and touching each other at their highest points If *APQ* and *Apq* be any two chords, prove that the times of descent down *PQ* and *pq* from rest at *P* and *p* are equal.

85. A tangent at any point *P* of a circle meets the tangents at the extremities of a vertical diameter *AB* in *C*, *D* respectively. If t_1 and t_2 be the times of sliding from rest down *CP and PD* respectively, prove that

$$\frac{t_1}{t_2}=\frac{\text{chord } AP}{\text{chord } BP}.$$

Chapter 6

Laws of Motion

6.1 Introduction

In the previous chapters, we have discussed about kinematics i.e. different kinds of motion of a body or a particle and the geometry of motion without any consideration of what has caused the motion or the mass that is moved. In this chapter, we shall discuss about kinetics or Dynamics proper i.e. the relations which exist between matter, force and motion. The motion of a body, the change in motion of a body when acted upon by an external force, the action of one body on another and so on can be investigated by three well-known laws of motion of Newton. In fact, the whole science of Dynamics is based on these three laws. Students have studied these laws with various illustrations in detail in physics in the earlier classes. We shall, therefore, explain these laws briefly in this chapter and derive important formulae.

6.2 Newton's Laws of Motion

There are three laws woich are stated below :

First Law · *Every body continues in its state of rest or of uniform motion in a straight line except in so far as it is compelled by some external impressed force to change that state.*

Second Law *The rate of change of momentum of a body is proportional to the impressed force, and takes place in the direction in which the force acts.*

Third Law · *To every action, there is an equal and opposite reaction.*

There is no direct proof of these laws. These laws can, therefore, be taken as the axioms.

6.3 First Law of Motion

There are three parts of the first law. The first part deals with the state of rest of a body, the second part, with the state of uniform motion in a straight line and the third part with the definition of force. The first part states that a body at rest cannot move on its own unless compelled by an external force to do so. The second part states that a body in motion will continue to move uniformly for ever in absence of any external force The third part of the law states that when there is a change of the state of rest or of uniform motion in a straight line or a tendency to have many of such changes, then the body must be acted upon by some force

The tendency of a body to continue in its state of rest or of uniform motion in absence of any external force is called the *property of inertia.* Hence, Newton's first law of motion is also known as *Law of Inertia. Inertia is thus a property which all matter possesses by virtue of which it resists any attempt to start it from rest or stop it if it is already in motion, or change in any manner its directian or motion.* The third part of Newton's first law of motion gives the definition of a force. Thus *a force is defined as that which changes or tends to change the state of rest or of uniform motion of a body in a straight line. Whenever we see that a body is at rest, it means that the resultant force acting on the body is zero.* Similarly, *when we find that a body is moving with a uniform velocity, we can conclude that there is no resultant force acting on it. Motion of a body with acceleration or retardation is due to the presence of a resultant force acting on the body.*

6.4 Second Law of Motion

Newton's second law of motion involves a physical quantity called "*Momemtum*" which is defined as follows ;

The momemtum of a moving particle or body at any time is defined as the product of its mass and its velocity at that instant. It is a vector quantity. Its direction is in the direction of the velocity. Its unit in S.I. system is kg m/s. Thus if m kg be the mass and v m/s be the velocity of a body at any time, then its momemtum is the mv kg m/s

While the first law of motion gives us information about *the existence of a force,* the second law gives *us the idea of measursng the force.*

6 5 Derivation of the Relation between Force, Mass and Acceleration

Let a continuous force F be acting on a body of mass m and let v be the velocity and a be the acceleration of the particle at any instant during the action of the force.

Since the body starts from rest, its initial momentum $=0$. The momentum of the body after time $t = m\,v$.

Hence, the change in momentum of the body during time $t = m\,v$.

$\therefore$ the rate of change of momentum of the body $= \frac{d}{dt}(mv)$

$= m\frac{dv}{dt}$, if m is taken to be constant.

By Newton's second law of motion,

$$F \propto m\frac{dv}{dt}$$

$$\therefore \quad F \propto ma$$

$$\Rightarrow \quad F = Kma \qquad \cdots (1)$$

where K is a constant.

Let us define a unit force as that force which acting on a unit mass produces a unit acceleration.

Thus, when $m=1$, $a=1$, we have $F=1$.

$\therefore \quad K=1$

Hence, from (1), we get

$$F = ma \qquad \cdots (1)$$

In other words, *the force acting on a body is given by the product of the mass and the acceleration of the body at that instant.*

Remark : The above formula, $F=ma$, is not applicable in the case of the motion of a body whose mass continually changes. For example, for the motion of a falling raindrop (on which aqueous vapour continuously accumulates and makes it bigger in size and hence in mass), the above formula is not applicable.

The equation of motion of such bodies with variable masses will be different from the above equation. The study of the motion of such bodies is beyond the scope of this book.

6.6 S I System of Force

The S. I. system of force, as stated in the author's book on "Elementary Statics" is *Newton.* A Newton is that force which acting on a mass of 1 kg, produces in it an acceleration of 1 m/s^2.

While using the formula, $F=ma$, in solving related problems, care should be taken to take the unit of F as Newton (N), m as kilogram (kg) and a as m/s^2.

6.7. Weight of a Body

The weight of a body is the force with which it is attracted by the earth. Since every falling body moves towards the earth with a uniform acceleration g, the weight w of a body of mass m kg is given by the formula $F=ma$.

Thus $$W=mg.$$

Thus the weight of a body is given by the product of its mass in kg and the acceleration g due to gravity in m/s².

If W_1 and W_2 be the weights of two bodies of masses m_1 and m_2 respectively at a place on the earth, then $W_1=m_1g$ and $W_2=m_2g$.

Hence, $$\frac{W_1}{W_2}=\frac{m_1}{m_2}.$$

In other words, *the weights of two bodies at a place are proportional to their masses.*

6.8 Principle of Physical Independence of Forces

Newton's second law of motion states that the effect on a body (i.e. change of momentum, or acceleration) due to a force is produced along the line of force In other words, *the effect of a force on a body will be produced in its own direction under all circumstances, whether the body is at rest or has an initial velocity in some other direction, or is acted on by other forces.* This principle is known as *the principle of physical independence of forces*

As an illustration, it may be observed that when a ball is let fall from the hand of a passenger in a moving train, the ball hits the floor of the train at exactly the same place where it would have fallen if the train were at rest. This shows that the ball has moved forward with the same velocity as that of the train The weight of the ball only changed motion in the vertical direction and did not affect the horizontal velocity of the ball.

6.9 Parallelogram of Forces

Since a force is a vector quantity, the resultant of two forces acting on a particle can be found out by *parallelogram of forces.* This is stated as follows

If a particle be acted on by two forces represented in magnitude and direction by two adjacent sides of a parallelogram drawn through a point, their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from that point.

The analytical derivation of the resultant of two forces is discussed in the author's book on "Elementary Statics".

6.10 Newton's Third Law of Motion

Newton's third law of motion gives us an idea about the *nature of forces*. If a body A exerts a certain force on another body B, then the third law implies that B exerts on A a force of equal magnitude which acts in opposite direction. The force exerted by A on B is called "action" and that exerted by B on A is called "reaction".

6.11 Pressure of a Body resting on a Moving Horizontal Plane

We consider a body of mass m resting on a horizontal plane. There are two cases to be considered : (i) When the horizontal plane is rising vertically upwards with an acceleration a and (ii) when the horizontal plane is descending vertically downwards with an acceleration a.

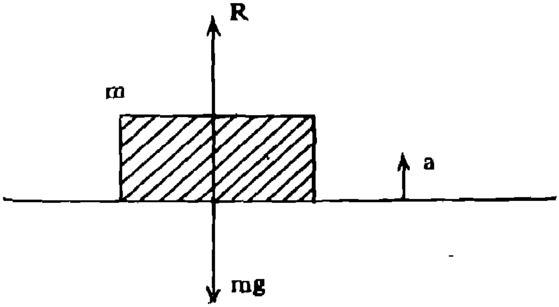


Fig. 49

In case (i), the upward acceleration of the body of mass m is a and the force which causes it to rise up is the resultant of R, the normal upward reaction of the plane on the body and the weight mg, of the body which acts vertically downwards.

Now, the resultant of the forces R and mg is $R-mg$. Hence, by Newton's second law of motion, we get

$$R-mg=ma$$

$$\Rightarrow \quad R=m(g+a) \qquad \ldots(1)$$

Now, by Newton's third law of motion, the pressure exerted by the body on the plane is equal and opposite to the reaction of the plane on it Hence, when the plane rises up, the pressure exerted by the body is given by (1).

We note that the pressure exerted by the body on the plane is more than its actual weight when the plane moves up. This is the reason why a man in a rising lift feels himself heavier than his actual weight.

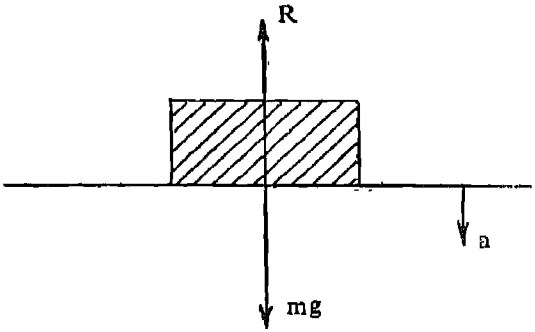


Fig. 50

Now, in the second case when the horizontal plane is descending vertically with acceleration a, the body of mass m moves down with the same acceleration. The resultant downward force acting on the mass m is then $mg-R$.

Hence, by Newton's second law of motion, we have

$$mg-R=ma$$

$$\Rightarrow \quad R=m\,(g-a)$$

By Newton's third law of motion, the pressure exerted by the body on the plane, being equal and opposite to the reaction R of the plane on the body is given by (2)

We note that the pressure exerted by the body on the plane is less than its actual weight when the plane moves down. This is the reason why a man in a descending lift feels himself lighter than his actual weight

Remark 1 : If the plane be at rest or if it moves up or down with constant velocity, then $a=0$ In this case, the pressure of the body on the plane will be equal to mg i.e. the weight of the body.

Remark 2 : In the case of upward motion of the plane, if the plane moves up with retardation i.e. if its velocity gradually diminishes as the plane moves up, then the pressure of the body on the plane will be less than its weight mg. Similarly, if the plane moves down with retardation i.e with diminishing velocity, its acceleration will be positive upwards and hence the pressure of the body on the plane will be greater than its actual weight mg

Remark 3 . If the plane moves down freely under gravity i.e. with acceleration equal to g, then $a = g$ so that $R = 0$. Hence, in this case, there will be no pressure of the body on the plane. Similarly, if the plane moves up with retardation equal to g, i.e. if $a = -g$, then $R = 0$ so that in this case also the pressure of the body on the plane will be zero.

6.12 Solved Examples

Ex 1. A mass m kg is acted on by a constant force of PN under which it moves a distance x m from rest in t s and acquires a velocity of v m/s. Show that $x = \frac{1}{2} \cdot \frac{mv^2}{P}$.

Acceleration produced

$$= a = \frac{P}{m} \text{ m/s}^2.$$

Now, using the formula $v = u + at$ and $s = ut + \frac{1}{2}at^2$, we get

$$v = at = \frac{Pt}{m} \qquad \ldots(1)$$

and

$$x = \frac{1}{2}at^2 = \frac{1}{2}\frac{Pt^2}{m} \qquad \ldots(2)$$

From (1), $t = \frac{mv}{P}$.

Substituting in (2), we get

$$x = \frac{1}{2} \cdot \frac{P}{m} \cdot \frac{m^2v^2}{P^2} = \frac{1}{2} \cdot \frac{mv^2}{P}.$$

Ex 2. A uniform force of 520×10^{-5} N acting for half a minute changes the velocity of a body moving in a straight line from 290 cm/s to 3.5 m/s. Find the mass of the body.

Let a m/s² be the acceleration produced and m kg be the mass of the body.

Then $$a = \frac{\text{Force}}{\text{mass}} = \frac{520 \times 10^{-5}}{m}$$

Initial velocity, $u=290$ cm/s$=2.9$ m/s.

Final velocity, $v=3.5$ m/s

Time during which the force acts$=\frac{1}{2}$ minutes$=30$ s

$\therefore$ using the formula $v=u+at$, we get

$$3.5=2.9+\frac{520\times10^{-5}}{m}\times30$$

$$\Rightarrow \quad .6\,m=156\times10^{-3}$$

$$\Rightarrow \quad m=\frac{156\times10^{-2}}{6}=26.$$

Hence, the required mass of the body is .26 kg or 260 g

Ex. 3. A mass of 4 kg falls 200 m from rest and is then brought to rest by penetrating 2m into some mud. Find the average thrust of the mud on it.

Let v m/s be the velocity on reaching the surface of mud Using the formula

$v^2=u^2+2gh$, we get

$$v^2=2\times9.8\times200=3920.$$

Let PN be the average thrust of the mud in the upward direction. Then the resultant upward thrust acting on the body is $(P-4g)$ N, since the weight of the body acts vertically downwards Let a m/s^2 be the retardation produced by this force The final velocity of the mass becomes zero while passing through the mud Hence, using the formula

$v^2=u^2-2as$, we get

$$0=3920-2a\times2$$

$$\Rightarrow \quad a=980.$$

Now, using the formula $F=ma$, we get

$$P-4g=4\times980$$

$$\Rightarrow \quad P=4\times980+4\times9.8$$

$$=3920+39.2$$

$$=3959.2$$

Hence, the required average thrust$=3959.2$ N.

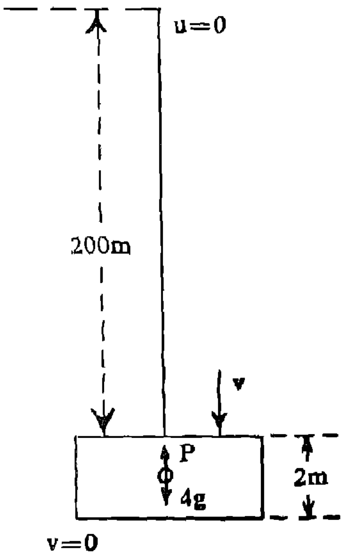


Fig. 51

Ex. 4 A body whose true mass was 13 kg appeared to weigh 12 kg when weighed by means of a spring balance in a moving lift. What was the acceleration of the lift at the innstant of weighing ?

Since the apparent weight of the body is less than its actual weight the lift was moving downwards. Let the downward acceleration of the lift be a. The apparent weight of the body is 13 $(g-a)$ N$=$13 $(9.8-a)$ N.

Hence, $13\,(9.8-a)=12\times9.8$

$$\Rightarrow \quad 127.4-13a=12\times9.8=117.6$$

$$\Rightarrow \quad 13a=9.8$$

$$\Rightarrow \quad a\simeq.75.$$

Hence, the required acceleration is 0.75 m/s^2 nearly

Ex. 5. A train is moving on a horizontal rail road. Assuming the weight of the train (exclusive of the engine) to be 16×10^5 N and the resistance arising from friction, etc. to be 40×10^{-4} N per unit weight of the train, find the pull between the engine and the train (i) when it is uniform, (ii) when the velocity of the train is uniform and (iii) when it is moving with an acceleration of 2 m/s^2.

(i) Let PN be the pull of the engine. The resistance is equal to

$$40\times10^{-4}\times16\times10^5 \text{ N}$$

$$=6400 \text{ N}.$$

∴ The resultant force acting on the train$=(P-6400)$ N.

In case (ii), since the velocity of the train is uniform, the acceleration produced is zero.

$$\therefore \; P-6400=\frac{16\times10^5}{9.8}\times0$$

$$\Rightarrow \quad P=6400$$

Hence, the pull of the engine is 6400 N.

In case (iii),

$$P-6400=\frac{16\times10^5}{9.8}\times2$$

$$\Rightarrow \quad P=6400+\frac{32\times10^6}{98}$$

$$=6400+326530.6$$

$$=332930.6.$$

Hence, the required pull of the engine is 332930.6 N.

Ex. 6 A man is raising, by means of a rope, 14 kg of water in a bucket of mass 3 5 kg, and he feels a uniform pressure of 210 N on his hand If the depth of the well be 24m, find the time he takes to raise the water to the surface. Find also the pressure on the bottom of the bucket.

The pull on the bucket is equal and opposite to the pressure on the hand Hence, the pull on the bucket=210 N. Let a be the acceleration with which the bucket ascends. The total vertically downward force acting on the bucket$=(14+3.5)\times 9\ 8$ N$=171.5$ N.

Hence, we have

$$210-171.5=17.5\,a$$

$$\Rightarrow \quad 38\ 5=17.5\,a$$

$$\Rightarrow \quad a=\frac{385}{175}=2.2.$$

Let t seconds be the required time. Using the formula

$s=ut+\frac{1}{2}at^2$, we get

$24=\frac{1}{2}\times 2\ 2\times t^2$ (Initial velocity of the bucket is assumed to be zero)

$$\Rightarrow \quad t^2=\frac{48}{2.2}=\frac{480}{22}=\frac{240}{11}\simeq 81.3182$$

$$\therefore \quad t\simeq 4.67.$$

Hence, the required time is 4 67 seconds nearly.

To find the pressure on the bottom of the bucket, we consider the motion of water only. Let R N be the reaction of the bucket on the water. The pressure of water on the bucket will also be R N. Hence, using the formula $F=ma$, we get

$$R-14\times 9.8=14\times 2.2$$

$$\Rightarrow \quad R=137\ 2+30.8$$

$$=168.$$

Hence, the required pressure of water on the bucket is 168 N.

Ex. 7. The velocity generated by a gun in bullet of $\frac{1}{32}$ kg is 300 m/s Assuming that the bullet described the length of the barrel in $\frac{1}{10}$-th of a second and that the force is uniform, find the acceleration and the moving force.

Let a m/s^2 be the required acceleration.

Using the formula $v=u+at$, we get

$$300=a\times\frac{1}{10} \Rightarrow a=3000.$$

Hence, the recuired acceleration is 3000 m/s^2.

Let F N be the required force.

Using the formula $F=ma$, we get

$$F=\frac{1}{32}\times 3000 \text{ N}$$
$$=93.75 \text{ N}.$$

Ex. 8 A train runs from rest for 1 km down an incline of 1 in 100. If the resistance be equal to .04 N per kg, how far will the train be carried along the horizontal at the foot of the incline ?

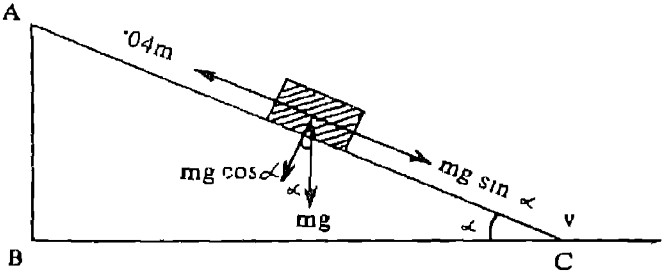


Fig 52

Let α be the inclination of the plane ABC so that $\sin\alpha=\frac{1}{100}$. Let m kg be the mass of the train. Then the force of resistance acting on the train up the plane $=.04\,m$ N. The component of the vertical weight mg down the plane is $mg\sin\alpha$ N$=\frac{mg}{100}$N.

Let a m/s² be the acceleration of the train down the plane. Then using the formula $F=ma$, we get

$$\frac{mg}{100}-.04\,m=ma$$

$$\Rightarrow \qquad a=\frac{9.8}{100}-.04=.098-.04=058.$$

Hence, the acceleration down the inclined plane=.058 m/s². Let v m/s be the velocity of the train at the bottom of the incline. Then considering the motion of the train on the plane $AC=1$ km$=1000$ m and using the formula $v^2=u^2+2\,as$, we get

$$v^2=2\times 058\times 1000=116 \qquad \ldots(1)$$

Now, when the train moves on the level ground, the only force opposing the motion is the force of resistance which is equal to $.04m$ N. Hence, using the formula $F=ma$ again, we get

$$.04\,m=ma' \quad \Rightarrow \quad a'=.04,$$

where a' is the retardation on the level ground. Hence, the retardation on the level ground= 04 m/s². Let the velocity of the train become zero after it describes a distance of x m. Hence, using the formula $v^2=u^2-2a'\,s$, we get

$$0=116-2\times .04\times x \qquad \text{[from (i)]}$$

$$\Rightarrow \qquad x=\frac{116}{08}=1450.$$

Hence, the required distance=1450 m or 1.450 km.

Ex 9. A bullet of mass $\frac{1}{32}$ kg is fired horizontally with an initial speed of 450 m/s and pierces a screen 180 m away after travelling for $\frac{1}{2}$ s. Find the resistance of the air, supposed constant.

The initial speed of the bullet=450 m/s.

Distance traversed by the bullet before piercing the screen=180 m.

Time during which the bullet travels$=\frac{1}{2}$ s

Let a m/s² be the retardation produced by the resistance of the air. Then using the formula $s=ut-\frac{1}{2}\,at^2$, we get

$$180=450\times\frac{1}{2}-\frac{1}{2}a\left[\frac{1}{2}\right]^2$$

$\Rightarrow$ $a=360.$

Hence, the resistance of the air

$$=R=ma=\frac{1}{32}\times 360\text{ N}=11.25\text{ N}.$$

Ex 10. A string can just support a mass of 10 kg when at rest. Show that the string will break if it raises a mass of 8 **kg** with an acceleration greater than 2.45 m/s^2.

The maximum value of the tension of the string is

$T_{max}=10\times 9.8$ N$=98$ N. When the mass is moving up with an acceleration, say a, then the equation of motion is

$$T-mg=ma,$$

where T is the tension of the string and $m=8$ kg.

Hence, $S=8\,(9.8+a)$

Now, $T\leqslant T_{max}$

$\Rightarrow$ $8\,(9.8+a)\leqslant 98$

$\Rightarrow$ $9.8+a\leqslant 12.25$

$\Rightarrow$ $a\leqslant 2.45$

Hence, the string will break if the acceleration exceeds 2.45 m/s^2

Ex 11. A thin glass plate can just support a weight of 13.5 kg. A body is placed on it and the plate is raised with the body on it with a gradually increasing acceleration It is found that the plate breaks when the acceleration is 1.2 m/s^2 Find the mass of the body.

The plate breaks when the acceleration a is 1.2 m/s^2. Thus the pressure on the plate is 13.5×9.8 N$=132.3$ N. If R be the upward reaction of the plate on the body, then

$$R=132.3\text{ N}.$$

Let m kg be the mass of the body. Considering the upward motion of the body, we have

$$R-mg=ma$$

$\Rightarrow$ $132.3=m\,(1.2+9.8)=11\,m$

$\Rightarrow$ $m\simeq 12.03$

Hence, the mass of the body is nearly 12.03 kg.

EXERCISE 6

1. An empty truck whose mass is 2000 kg has a maximum acceleration of 1 m/s^2. What will be its maximum acceleration when it carries a load of 1000 kg ?

[Ans. $\frac{2}{3}$ m/s^2]

2. A 1000 kg car goes from 10 m/s to 20 m/s in 5 *s*. What force is acting on it ? [Ans. 2000 N]

3 The brakes of a 1000 kg car exert a force of 3000 N (i) How long will it take the car to come to a stop from a velocity of 30 m/s ? (ii) How far will the car travel during this time ?

[Ans. (i) 10 s (ii) 150 m]

4. While launching a rocket of mass 2×10^4 kg, a force of 5×10^5 N is applied for 20 s What is the velocity attained by the rocket at the end of 20s [D.B S.S C '84 (c)]

[Ans. 500 m/s]

5 A car of mass 1000 kg is moving up at a velocity of 10 m/s and is acted by a forward force of 1000 N due to engine and a retarding force of 500 N due to friction What will be its velocity after 10 s ? [A.I.S S C, '81]

[Ans. 15 m/s]

6. A 2.5 kg body at rest is acted upon by a constant external force of 10 N until a displacement of 30 m is obtained. What is the speed attained by the body after this displacement ?

[Ans. 15 5 m/s approx.]

7 A bullet of mass 0.01 kg is fired with a velocity of 800 m/s. After passing through a mud wall, 1 m thick, its velocity drops to 100 m/s. Calculate the average resistance of the wall. Neglect the friction due to air. [Ans 3150 N)

8. Find the weight of 6 kg of potatoes and find the mass of 6 N of potatoes. [Ans. 59 N. approx ; 0.61 kg approx]

9. A force of 10 N is applied to (i) a body of mass 5 kg and (ii) a body of weight 5 N. Find their accelerations.

[Ans. (i) 2 m/s^2 (ii) 19.6 m/s^2]

10. How much upward force is needed (i) to support a 20 kg object

at rest (ii) to give it an upward acceleration of 2 m/s^2 and (iii) to give it a downward acceleration of 2 m/s^2 ?

[Ans. (i) 196 N (ii) 236 N (iii) 156 N]

11 What is the acceleration of a 5 kg object suspended by a string when (i) an upward force of 39 N is applied to the string, (ii) an upward force of 49 N is applied to the string and (iii) an upward force of 59 N is applied to the string ?

[Ans. (i) 2 m/s^2 downwards (ii) 0 m/s^2 (iii) 2 m/s^2 upwards]

12. A certain force gives a 2 kg object an acceleration of 0.5 m/s^2. What acceleration would the same force give a 10 kg object ? [Ans. 0.1 m/s^2]

13. How much force is needed to accelerate a train whose mass is 10^6 kg from rest to a velocity of 6 m/s in 2 minutes ?

[Ans. 5×10N]

14. A 0.05 kg snail goes from rest to a velocity of 0.01 m/s in 5 s. How much force does it exert and how far does it go during this time ? [Ans. 10^{-4} N , .025 m]

15 A force of 6 N acts for 2 seconds on a mass of 3 kg which is originally moving with a velocity of 10 m/s in the direction of the force. Calculate the final velocity and the change in momentum of the body. [Ans. 14 m/s ; 12 kg m/s]

16. A mass of 10 kg is travelling with a velocity of 3 m/s when a constant force is applied for 4s in the opposite direction. At the end of this time the velocity of the body is -2 m/s. Calculate the magnitude of the applied force.

[Ans. 12.5 N in the opposite direction to the original velocity]

17. A mass of 10 kg has a velocity of 20 m/s. A force of 100 N is applied to the mass in the direction of motion and increases the velocity to 35 m/s. Find the time for which the force is applied.

[Ans. 1.5 s]

18 A car which has a total mass (including passengers) of 700 kg increases velocity from 14 km/h to 50 km/h in 35 s. Calculate the constant force which has been applied. [Ans. 200 N]

19. A mass of 4 kg falls freely from rest at a height of 4.9 m. Calculate the momentum of the body when it hits the ground.

[Ans. 39.2 kg m/s]

20. A mass of 2 kg hits the ground with a velocity of 10 m/s and rebounds in the same vertical line with a velocity of 5 m/s.

Calculate the change in momentum of the body. [Ans 30 kg m/s]

21. A body acted on by a uniform force in 10 s describes a distance of 7 m Compare the force with the weight of the body and find the velocity. [Ans. 1 : 70 ; 1.4 m/s]

22. A mass of 5 kg falls 4 m from rest and is then brought to rest by penetrating 20 cm into some mud Find the average thrust of the mud [Ans 1029 N]

23 In a diving competition, the board is fixed at a height of 10 m above the water level. A competitor jumps from the board and dives to a depth of 5 m Determine the resistance offered by the water. Assume the weight of the competitor to be 60 kg [Ans 1764 N]

24. A heavy body of mass 64 kg is being raised form the bottom of a pit, 30 m deep, with s uniform force of 784 N. Find the time taken by the body to reach the top of the pit [Ans. 4.95 s nearly]

25. Each of two bodies at rest on a smooth horizontal table attracts the other with the same force irrespective of the distance between the two bodies Show that in any time, they move over the spaces which are inversely as their masses. If the masses of the two bodies be 4.5 kg and 8 kg, the constant force is 5 N and the distance between the masses be 10 m, find after what time they would meet [Ans. 3.34 s nearly]

26. If out of two forces, one acts on a mass of 10 kg and in one-fifth of a second produces in it a velocity of 2 m/s and the other acting on a mass of 625 kg in a minute produces in it a velocity of 18 km/h. Compare the two forces. [Ans. 48 : 25]

27 A bullet moving at the rate of 120 m/s is fired into a trunk of wood into which it penetrates 30 cm. If a bullet moving with the same velocity were fired into similar piece of wood 25 cm thick, with what velocity would it emerge, supposing the resistance to be uniform ? [Ans. 48.99 m/s]

28. An elevator weights 4000 kg when the upward tension in supporting cable is 48000 N. What is the upward acceleration ? How far does it rise in 3 s if it starts from rest ? [Ans 2.2 m/s^2 ; 9 9 m]

29. A thief jumps off the terrace of a building with a heavy load on his head, and falls vertically. What would be the pressure of the load on his head while he is falling ? [Ans. 0 N]

30. A 80 kg man stands on a scale in an elevator car. When it starts to move, the scale reads 700 N (i) Is the car moving upwards or downwards ? (ii) Is the velocity constant ? (iii) If so, what is it ? If not, what is its acceleration ?

[Ans. (i) Downwards (ii) No (iii) 1.05 m/s]

31. A balloon whose total mass is M is falling with downward acceleration a_1. Neglecting frictional resistance due to the air, find how much ballast should be ejected from the balloon in order that it should then have an upward acceleration a_2

$$\left[\text{Ans. } \frac{M(a_1+a_2)}{g+a_2}\right]$$

32. A engine exerts a force of 45×10^3 N on a train weighing 25×10^5 N and draws it up a slope of 1 in 100 against resistance of 1750 N. Find
   (i) the acceleration of the train
   (ii) the breaking force that would be required on the return journey to prevent the acceleration exceeding .01 m/s^2

[Take $g_2=10$ m/s^2]

[Ans (i) 0.073 m/s^2 (ii) 20750 N]

33. It was found that when 40 cm were cut off from the muzzle of a gun firing a projectile of 40 kg, the velocity was altered from 550 m/s to 431 m/s Show that the force exerted on the projectile by the powder gas at muzzle when expanded in the bore was 5836950 N.

34. A spring balance loaded with a body of mass 3 kg reads 2 8 kg when suspended inside a moving lift. What can be inferred as to the motion of the lift and what is its acceleration ?
[Ans. The lift has a downward acceleration of 0 65 m/s^2 nearly]

35. A cannon ball of mass 1 kg is discharged with a velocity of 450 m/s from a cannon, the length of whose barrel is 2 m ; show that the mean force exerted on the ball during the explosion is 5×10^4 N nearly.

36. A passenger lift of mass 1524 kg is supported by a wire rope. Determine the tension in the rope when the lift is (i) moving up with uniform velocity (ii) moving up with an acceleration of 2 m/s^2 (iii) moving down with an acceleration of 3 m/s^2.

[Ans. (i) 14935.2 N (ii) 17980 N nearly (iii) 10368 N nearly]

37. A scale pan on which rests a mass of 0.005 kg is drawn upwards with a constant acceleration and the reaction between the mass and the pan is found to be 0.5 N Find the acceleration of the scale pan [Ans. 90.2 m/s^2]

38. A body of mass 2 kg is falling vertically under gravity at the rate of 50 m/s. What is the uniform force vertically upwards that will stop it (i) in 2 s (ii) in 2 m. [Ans. (i) 69.60 N (ii) 1269.6 N]

39. A boy whose mass is 50 kg stands on a spring weighing machine inside a lift. The lift starts to ascend with an acceleration of 2.45 m/s^2. What is the reading in the machine ? What will be the reading in the machine when in its ascent, the lift moves (i) with uniform velocity, (ii) with a retardation of 4.5 m/s^2. [Ans. 62.5 kg (i) 50 kg (ii) 27.04 kg]

40. A boy with a basket of 4 kg of sweets hanging from his hand is descending in a lift. The lift starts down with an acceleration of 6 m/s^2, reaching a steady speed which it keeps up till it slows down at the rate of 1.2 m/s^2. Find the pressure on the hand of the boy during the three stages of the descent of the lift. [Ans. 15.2 N ; 39.2 N ; 44N]

41. A load, W, is raised by a rope, from rest to rest, through a height h ; the greatest tension which the rope can safely bear is $n W$ Show that the least time in which the ascent can be made is $\left(\frac{2nh}{(n-1)g}\right)^{\frac{1}{2}}$

42. A railway train of mass m kg goes from rest at one station to rest at another in t s, The distance between the stations is d m and resistance of the rails $a\,g$ N. If the engine exerts a constant tractive force of Pg N for a part of the distance. show that $P=\frac{a^2 gt^2}{agt^2-2md}$ and that P acts for the time $\left(t-\frac{2md}{2gt}\right)$ s

Chapter 7

Application of Newton's Second Law in the Motion of Connected Systems

7.1 Introduction

In this chapter, we shall consider the motion of a system of bodies connected by one or more strings In solving problems of this type, we shall consider the motion of each body separately For this, all external and internal forces acting on each moving body should be found out and their directions should be indicated in the diagram, From the direction of motion of a body, we find the direction of the resultant force and hence write down the dynamical equation of motion, $F=ma$, for each body of the system. We shall thus obtain a number of equations from which the unknown quantities can be found out. We consider below the motion of a few systems of bodies and explain the procedure.

7.2 Motion of Two Connected by a String passing over a Pulley when both the Particles hang freely

Two bodies of masses m_1 *and* m_2 $(m_1>m_2)$ *are connected by a light inextensible string passing over a light smooth pulley. To find (i) the acceleration of two systems (ii) the tension in the string and (iii) the pressure on the pulley.*

As $m_1>m_2$, the body of mass m_2 will move up and that of mass m_1 will move down. Since the string is inextensible, the upward displacement of m_2 will be equal in magnitude to the downward displacement of m_1 at every instant of motion of the bodies. Since the rate of change of displacement is the velocity, hence the upward velocity of m_2 at every instant will be equal in magnitude to the downward velocity of m_1. Hence, the acceleration which is the rate of change of velocity will be the same

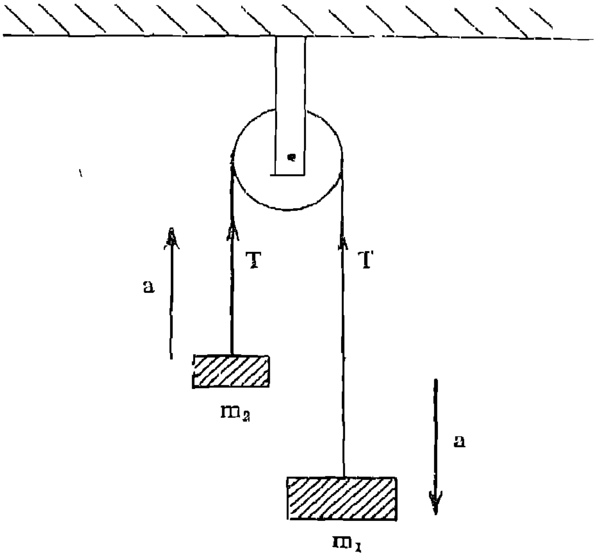


Fig. 53

in magnitude for each of the two bodies. Let a be the acceleration of each body. Since the same string passes over the pulley and the string is light, the tension of the string on each side of the pulley will be the same throughout its length Also, since the pulley is light and smooth, the tension does not change along the string as it passes from one side to the other over the pulley. In other words, the tension of the string is constant throughout. Let T be the tension of the string

Considering the downward motion of the mass m_1, the forces acting on it are its weight m_1g downwards and the tension T upwards Since the acceleration of m_1 is a in the downward direction, the equation of motion of the mass m_1 is

$$m_1g-T=m_1a \qquad \text{.. (1)}$$

Similarly, considering the upward motion of the mass m_2, we get

$$T-m_2g=m_2a \qquad \text{.. (2)}$$

Adding (1) and (2), we get

$$(m_1-m_2)g=(m_1+m_2)a$$

$$\Rightarrow \qquad a=\frac{m_1-m_2}{m_1+m_2}\,g \qquad (3)$$

Substituting the value of a in (1), we get

$$m_1g-T=\frac{m_1(m_1-m_2)g}{m_1+m_2}$$

$$\Rightarrow \qquad T=m_1g-\frac{m_1(m_1-m_2)g}{m_1+m_2}$$

$$=\frac{m_1^2g+m_1m_2g-m_1^2g+m_1m_2g}{m_1+m_2}$$

$$=\frac{2m_1m_2}{m_1+m_2}\,g. \qquad (4)$$

As the string presses the pulley downwards on both sides, the pressure on the pulley will be the resultant of two equal and like parallel forces T, T. Hence, the pressure on the pulley

$$=2\,T=\frac{4m_1m_2}{m_1+m_2}\,g \qquad .\,(5)$$

To find a relation between T and a when m_1+m_2 remains constant, we have from (3),

$$a^2=\frac{(m_1-m_2)^2}{(m_1+m_2)^2}\,g^2$$

$$=\frac{(m_1+m_2)^2-4m_1m_2}{(m_1+m_2)^2}\,g^2. \qquad ..\,(A)$$

From (4), we have

$$2m_1m_2=\frac{(m_1+m_2)T}{g}$$

Hence, from (A)

$$a^2=\frac{(m_1+m_2)^2-\dfrac{2(m_1+m_2)T}{g}}{(m_1+m_2)^2}\,g^2$$

$$=g^2-\frac{2Tg}{(m_1+m_2)} \qquad ...(6)$$

We observe from (6) that as T increases, a decreases and vice versa provided m_1+m_2 is constant.

7.3 Motion of Two Bodies Connected by a String Passing over a Pulley when one of the Particles rests on a Table

Two bodies of masses m_1 and m_2 are connected by a light inextensible string passing over a light smooth pulley at the edge of a smooth plane, m_2 lying on the table and m_1 hanging vertically.

We shall consider two cases .

Case 1 : *When the smooth plane is horizontal.*

Let a be the acceleration with which the mass m_1 descends. As the string is inextensible, the mass m_2 will move on the table along the string with the same acceleration Let R be the normal reaction of the plane on the mass m_2. Let T be the tension of the string, which will be constant throughout its length, since the string is light and inextensible and the pulley is light and smooth

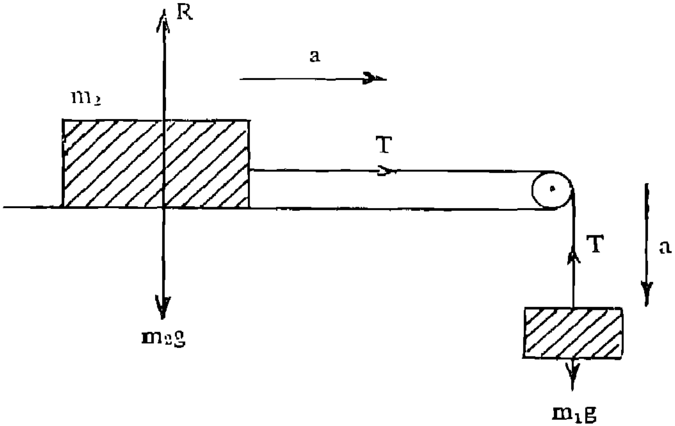


Fig. 54

Now, considering the motion of m_1 downwards, we have

$$m_1 g - T = m_1 a. \qquad \ldots(1)$$

Considering the horizontal motion of m_2 on the table, we get

$$T = m_2 a \qquad \ldots(2)$$

Adding (1) and (2), we get

$$m_1 g = a(m_1 + m_2)$$

$$\Rightarrow \qquad a = \frac{m_1 g}{m_1 + m_2} \qquad .\ (3)$$

Hence, from (2), we get

$$T = \frac{m_1 m_2 g}{m_1 + m_2}. \quad \ldots(4)$$

Since there is no vertical motion of the mass m_2, we have

$$R = m_2 g \quad \ldots(5)$$

which gives the pressure of the particle on the table.

To find the pressure on the pulley, we find that it is the resultant of two tensions T, T of the two parts of the string pressing the pulley in perpendicular directions. Hence, the pressure on the pulley

$$= \sqrt{T^2 + T^2} = \sqrt{2}\, T$$

Case 2 : *When the smooth plane is inclined.*

Let the inclination of the plane be α. Here we assume that $m_1 > m_2 \sin\alpha$. Then the resolved part of $m_2 g$ down the plane i.e. $m_2 g \sin\alpha$ will be less than $m_1 g$ so that the mass m_1 will move downwards and draw m_2 up the plane.

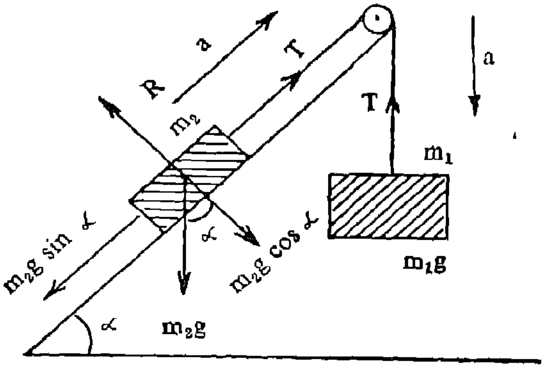


Fig. 55

Considering the vertically downward motion of m_1, we get

$$m_1 g - T = m_1 a.$$

Again, considering the motion of m_2 up the plane, we get

$$T - m_2 g \sin\alpha = m_2 a. \quad \ldots(7)$$

Adding (6) and (7), we get

$$g(m_1-m_2 \sin \alpha)=(m_1+m_2)a$$

$$\Rightarrow \qquad a=\frac{(m_1-m_2 \sin \alpha)\, g}{m_1+m_2}. \tag{8}$$

Multiplying (6) by m_2 and (7) by m_1 and subtracting, we get

$$m_2m_1g-m_2T-m_1T+m_1m_2g \sin \alpha=0.$$

$$\Rightarrow \qquad T(m_1+m_2)=m_1m_2g\,(1+\sin \alpha)$$

$$\Rightarrow \qquad T=\frac{m_1m_2\,(1+\sin \alpha)}{m_1+m_2}\, g\,. \qquad \text{.. (9)}$$

Since there is no motion perpendicular to the plane, we have

$$R=m_2g \cos \alpha. \qquad \text{.(10)}$$

The pressure on the pulley will be the resultant of tensions T, T of the two parts of the string pressing the pulley and since the angle between these two equal forces is $\frac{\pi}{2}-\alpha$, the pressure on the pulley is

$$=\sqrt{T^2+T^2+2T^2 \cos\left(\frac{\pi}{2}-\alpha\right)}$$

$$=\sqrt{2}\,T\sqrt{1+\sin \alpha}=\sqrt{2}\,T\left(\cos\frac{\alpha}{2}\cdot\sin\frac{\alpha}{2}\right)$$

$$=2T \cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right).$$

Remark 1 : If $m_1<m_2 \sin \alpha$, we see from (8) that a is negative so that m_1 will rise upwards with acceleration $\frac{(m_2 \sin \alpha-m_1)g}{m_1+m_2}$ and m_2 will descend down the plane with the same acceleration.

Remark 2 . Putting $\alpha=\frac{\pi}{2}$, we get the result discussed in section 7.2 and putting $\alpha=0$, we get the result discussed in section 7.3, case 1, as particular cases of the above general result

7.4. Motion of Two Bodies Connected by a String Passing over a Pulley when the Bodies lie on Two Smooth Inclined Planes

Let the body of mass m_1 move down an inclined plane of inclination and the body of mass m_2 move up another inclined plane of inclination

β so that $m_1 \sin\alpha > m_2 \sin\beta$. Let R_1, R_2 be the normal reactions of the planes, a be the common acceleration and T be the tension of the string thoughout its length.

Considering the downward motion of m_1, we get

$$m_1 g \sin\alpha - T = m_1 a. \qquad \ldots(1)$$

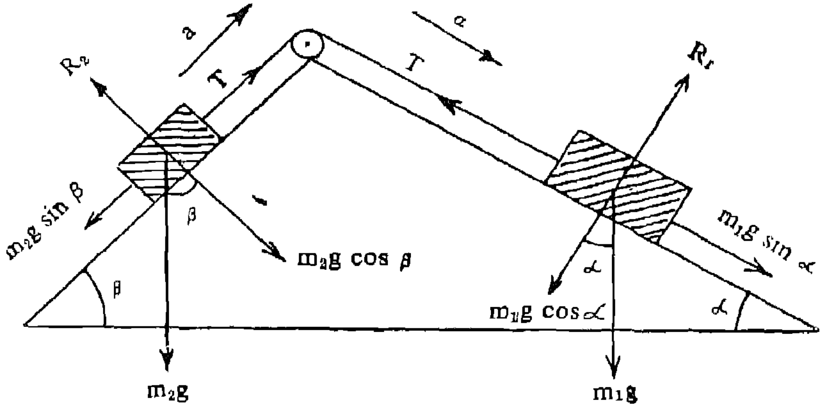


Fig. 56

Considering the upward motion of m_2, we get

$$T - m_2 g \sin\beta = m_2 a. \qquad (2)$$

Also, since there is no motion perpendicular to the planes, we have

$$R_1 = m_1 g \cos\alpha \qquad . \ (3)$$

$$R_2 = m_2 g \cos\beta. \qquad . \ (4)$$

Adding (1) and (2), we get

$$\Rightarrow \qquad a = \frac{(m_1 \sin\alpha - m_2 \sin\beta)\, g}{m_1 + m_2} \qquad (5)$$

Multiplying (1) by m_2 and (2) by m_1 and subtracting, we get

$$m_1 m_2 (\sin\alpha + \sin\beta)\, g - T\,(m_1 + m_2) = 0$$

$$\Rightarrow \qquad T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)\, g}{m_1 + m_2}. \qquad \ldots(6)$$

The pressure on the pulley will be the resultant of T, T inclined at an angle of $180° - (\alpha + \beta)$ with each other.

Hence, the pressure on the pulley

$$= \sqrt{T^2 + T^2 + 2T^2 \cos[180° - (\alpha + \beta)]}$$

$$= \sqrt{2}\, T \sqrt{1 - \cos(\alpha + \beta)} = 2\, T \sin\frac{\alpha + \beta}{2} \qquad \ldots(7)$$

Remark : If we put $\alpha=\frac{\pi}{2}$, all the results are identical with those discussed in case 2 of section 7.3

7.5. Solved Examples

Ex. 1 What acceleration must be imparted to a load hanging from a cord passing over a pulley in order that the tension in the cord may be twice the weight of the load.

If W be the load and T be the tension of the string, then $T=2W$. Let a be the acceleration with which this load is moving up due to some other load at the other end of the string, the string being supposed to pass over a smooth light pulley

Then the equation of motion of the load W is

$$T-W=\frac{W}{g}\,a$$

$$\Rightarrow \qquad 2W-W=\frac{W}{g}\,a$$

$$\Rightarrow \qquad a=g.$$

Hence, the required acceleration is equal to that due to gravity.

Ex. 2. Two particles of masses 7 kg and 5 kg are connected by a light inextensible string which passes over a small smooth fixed pulley Find the velocity at the end of 4 seconds and the distance described during this time.

Here the acceleration a of the system is given by

$$a=\frac{m_1-m_2}{m_1+m_2}\,g=\frac{7-5}{7+5}\times 9.8 \text{ m/s}^2$$

$$=\frac{2\times 9.8}{12} \text{ m/s}^2=\frac{49}{30} \text{ m/s}^2$$

Now, to find the velocity at the end of 4 s, we use the formula $v=u+at$ where $u=0$. Thus we get

$$v=\frac{49}{30}\times 4 \text{ m/s}=6\frac{8}{15} \text{ m/s}.$$

To find the distance described after 4s, we use the formula $s=ut+\frac{1}{2}at^2$ and obtain

$$s=\frac{1}{2}\times\frac{49}{30}\times 16m=13\frac{1}{15}m.$$

Ex. 3. Equal masses of .5 kg hang from the extremities of a string passing over a smooth pulley. If a mass of .02 kg be removed from one of the masses, find the time in which the vertical distance between them increases by 5 m.

Initially, the two equal loads, *A* and *B*, were at the same horizontal line *XY*. When .02 kg mass is removed from one of these two loads, say *A*, then the load *A* becomes lighter and comes to the position, say *A'* and hence *B* will come down to the position, say *B'*. Clearly, the distance traversed by *A* will be equal to that traversed by *B*. Hence, when the distance between the two loads is 5 m, each will traverse a vertical distance of $\frac{5}{2}$ m.

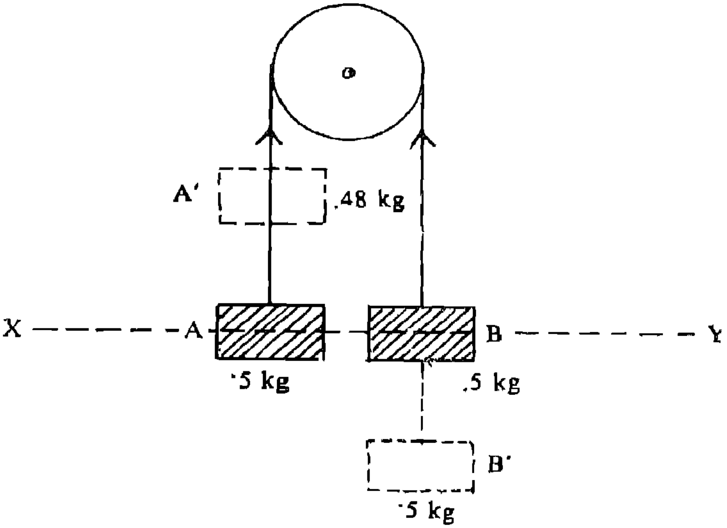


Fig. 57

Now, after taking away a mass of .02 kg from one of the loads, the two masses will be

$$m_1 = .5 \text{ kg}, \ m_2 = (5 - .02) \text{ kg} = .48 \text{ kg}$$

If a m/s^2 be the acceleration of the system, then

$$a = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{.5 - .48}{.5 + 48} \times 9.8$$

$$= \frac{.02 \times 9\ 8}{98} = .2.$$

Hence, the acceleration of the system is $.2$ m/s^2. If t s be the time after which the distance between the two loads is 5 m i e. during which each load describes a distance of $\frac{5}{2}$ m, each starting from rest, then we have

$$\frac{5}{2}=\frac{1}{2}\times .2\times t^2 \quad \left(\text{Using the formula } s=ut+\frac{1}{2}at^2\right)$$

$$\Rightarrow \quad t^2=25 \quad \Rightarrow \quad t=5.$$

Hence, the required time is 5 s

Ex. 4 Two masses are connected by an inelastic string passing over a smooth pulley and one mass is 5 times the other. After moving for 3 s from rest the descending mass is suddenly stopped and immediately allowed to fall again Find the time that elapses before the string becomes tight again

Let the mass of the two bodies be $m_1=m$ kg and $m_2=5\,m$ kg Then the acceleration a m/s^2 of the system is given by

$$a=\frac{m_2-m_1}{m_2+m_1}\,g=\frac{4\times 9.8}{6}=\frac{98}{15}.$$

The velocity v m/s of each mass after 3 s is given by

$$v=u+at$$

$$=\frac{98\times 3}{15}=\frac{98}{5}$$

The heavier mass is now suddenly stopped As a result, the string will become slack and the lighter mass will move freely under gravity in the vertically upward direction. Since the heaviour mass is immediately allowed to fall, it will fall freely under gravity. It should be noted here that at the beginning the smaller mass rises more than the heavier mass falls, but after sometime, the heavier mass falls more than the smaller mass rises If the string becomes tight after t s, then the distance fallen by the heavier mass in t s is equal to the distance risen by the smaller mass in the same time

Now, the distance fallen by the heavier mass $=\frac{1}{2}gt^2$ and the distance risen by the smaller mass $=vt-\frac{1}{2}gt^2$

$$=\frac{98}{5}t-\frac{1}{2}gt^2.$$

Hence, $$\frac{1}{2}gt^2=\frac{98}{5}t-\frac{1}{2}gt^2$$

$$\Rightarrow \quad gt^2=\frac{98}{5}t \quad \Rightarrow \quad t=\frac{98}{5}\times\frac{1}{98}=\frac{10}{5}=2$$

Hence, the required time is 2 s.

Remark : After 2 s, the velocity of the lighter mass is $\frac{98}{5}-9.8\times2=0$ so that we observe that the string becomes tight again when the velocity of the smaller mass becomes zero during its free motion in the upward direction.

Ex. 5. A mass of 5 kg descending vertically, draws up a lighter mass, by means of a thin string passing over a smooth pulley. At the end of 2 s, the string breaks. If the lighter body rises 1 m higher, find its mass.

Let m kg be the mass of the lighter body. If a m/s^2 be the acceleration of the system. Then

$$a=\frac{5-m}{5+m}g=\frac{(5-m)\,9.8}{5+m}.$$

Let v m/s be the upward velocity of the lighter body after 2 s just before the string breaks. Then using the formula $v=u+at$, where $u=0$, we get

$$v=\frac{(5-m)\,9.8\times2}{5+m}. \qquad (1)$$

When the string breaks, the lighter body moves up further freely under gravity with initial velocity v given by (1). If this body rises 1 m higher after the string breaks, then at the heighest point the velocity of the body will be zero. Hence, using the formula $v^2=u^2-2gh$, we get

$$0=\frac{(5-m)^2\,9.8^2\times2^2}{(5+m)^2}-2\times9.8\times1$$

$$\Rightarrow \quad \frac{(5-m)^2\times9.8\times2}{(5+m)^2}=1$$

$$\Rightarrow \quad (5-m)^2\times19.6-(5+m)^2=0.$$

$$\Rightarrow \quad (25-10m+m^2)\,19.6-25-m^2-10m=0$$

$$\Rightarrow \quad 18.6\,m^2-206m+465=0$$

$$\Rightarrow \quad m=\frac{206\pm\sqrt{206^2-4\times18.6\times465}}{37.2}$$

$$=\frac{206+\sqrt{7840}}{37.2}\simeq\frac{206\pm88.54}{37.2}$$

$$=\frac{2945.4}{372}\text{ or }\frac{1174\,6}{372}$$

$$=7.9\text{ or }3.2.$$

We reject $m=7.9$, since we consider the mass of the lighter body. Hence, the required mass of the lighter body is 3.2 kg nearly.

Ex. 6. A light rope hangs over a smooth pulley. A monkey of mass 5 kg climbs down the portion of the rope on one side with an acceleration of 1 m/s^2. Find with what acceleration another monkey of mass 4 kg will climb up the portion of the rope on the other side so that the rope may remain at rest.

Let TN be the tension of the string and let a m/s^2 be the acceleration of the monkey which is rising up Then the total force acting on this monkey $(T-4g)$ N.

$$\therefore\qquad T-4g=4a. \qquad ..\ (1)$$

Similarly, the equation of motion of the other monkey which is climbing down the rope with acceleration of 1 m/s^2 is

$$5g-T=5\times1=5. \qquad (2)$$

Adding (1) and (2), we get

$$g=5+4a$$

$$\Rightarrow\qquad \frac{9\,8-5}{4}=a$$

$$\Rightarrow\qquad \frac{4.8}{4}=a\quad\Rightarrow\quad a=1.2.$$

Hence, the required acceleration of the other monkey is 1.2 m/s^2.

Ex. 7. A string passing over a smooth fixed pulley supports at one end a mass m, and at the other a small light movable pulley over which passess a string upporting masses m_1, m_2 at its ends. If T, T' be the tensions of he strings respectively, prove that

$$\frac{2T}{m}+T'\left(\frac{1}{m_1}+\frac{1}{m_2}\right)=4g.$$

(Note that in solving problems of the above type where one or more strings pass over more than one pulley, representation of acceleration in terms of the derivative in the equations of motion of different bodies is onvenient We shall represent the acceleration like $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}$ ith the symbols $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, etc. respectively for convenience).

Let P and L be the fixed and light pulleys respectively. Let x m be of the mass m, y m be the depth of the light pulley L from the fixed y P and let z m and w m be the depths of masses m_1 and m_2 from the pulley L. Then $x+y=$constant.

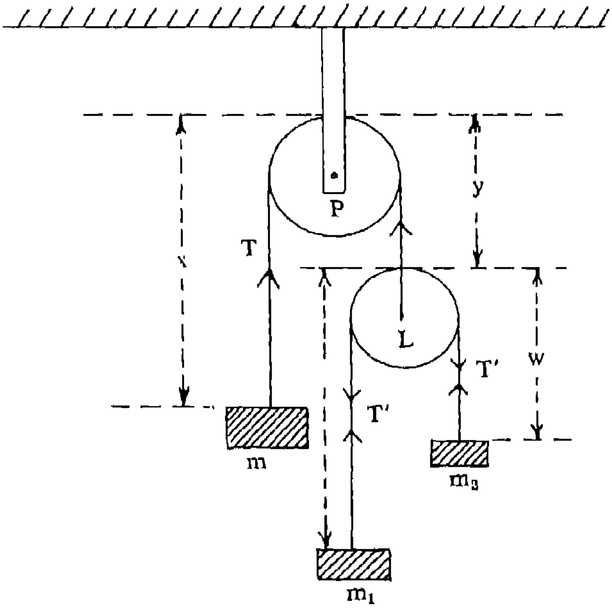


Fig. 58

$$\therefore \qquad \ddot{x}+\ddot{y}=0 \qquad \ldots(1)$$

Similarly,

$$z+w=\text{constant}.$$

$$\therefore \qquad \ddot{z}+\ddot{w}=0 \qquad \ldots(2)$$

Then the equation of motion of the mass m is $mg-T=m\ddot{x}$

$$\Rightarrow \qquad g-\frac{T}{m}=\ddot{x} \qquad \ldots(3)$$

The equation of motion of the mass m_1 kg which is at a distance $(y+z)$ m from the fixed pulley is

$$m_1 g - T' = m_1 (\ddot{y}+\ddot{z})$$

$$\Rightarrow \qquad g - \frac{T'}{m_1} = \ddot{y} + z \qquad (4)$$

Similarly, the equation of motion of the mass m_2 kg which is at a distance $(y+w)$ m from the fixed pulley is

$$m_2 g - T' = m_2 (\ddot{y} + \ddot{w})$$

$$\Rightarrow \qquad g - \frac{T'}{m_2} = y + \ddot{w} \ . \qquad \ldots(5)$$

Multiplying equation (3) by 2 and adding with equations (4) and (5), we get

$$2\ddot{x} + 2\ddot{y} + \ddot{z} + \dot{w} = 4g - \frac{2T}{m} - T'\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$\Rightarrow \qquad 0 = 4g - \frac{2T}{m} - T'\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

[by (1) and (2)]

$$\Rightarrow \qquad \frac{2T}{m} + T'\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = 4g.$$

Hence the result.

Remark : While writing down the equations of motion in the above problem, we have not considered the particular direction towards which a body is actually moving, since it is not necessary. The positive direction of the resultant force acting on a body should always be taken in that direction in which the distance of the body measured from some fixed point of the line increases during the motion of the body. Thus in the above problem, since the increasing direction of x, y, z is in the downward direction, the positive direction of the resultant force has been taken to be in that direction. In other words, we can assume as if all the masses are moving downwards.

Ex. 8. Two pulleys of masses 6 kg and 4 kg are connected by a fine string hanging over a smooth fixed pulley. Over the former is hung a fine string with masses 1.5 kg and 3 kg at its ends and over the latter a fine string with masses 2 kg and x kg Determine x so that the string over the fixed pulley remains stationary, and find the tension in it.

Let A be the fixed pulley, and B and C be the movable pulleys of masses 6 kg and 4 kg respectively. Let T N be the tension of the string passing over the fixed pulley and T_1 N, T_2 N be the tensions of the strings passing over the other two movable pulleys respectively. Let the depths of the movable pulleys measured from the fixed pulley be y_1 and y_2 respectively measured in metres. Let the depths of masses 1.5 kg, 3 kg, 2 kg and x kg from the fixed pulley be z_1, z_2, z_3 and z_4, all measured in metres

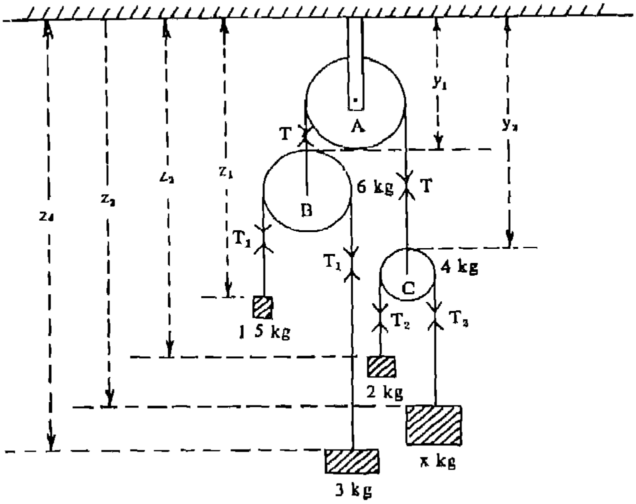


Fig 59

Now, since each of y_1+y_2, $z_1+z_2-2y_1$ and $z_3+z_4-2y_2$ is a constant, we have

$$\ddot{y}_1+\ddot{y}_2=0 \quad (1)$$

$$\ddot{z}_1+\ddot{z}_2-2\ddot{y}_1=0 \quad (2)$$

and
$$\ddot{z}_3+\ddot{z}_4-2\ddot{y}_2=0 \quad (3)$$

Since the string over the fixed pulley remains stationary, both y_1 and y_2 are constant.

Hence,

$$\ddot{y}_1=\ddot{y}_2=0 \quad (4)$$

Now, the equations of motion of the two movable pulleys are respectively

$$6g+2T_1-T=6\ddot{y}_1 \quad (5)$$

and
$$4g+2T_2-T=4\ddot{y}_2 \quad (6)$$

Also, the equations of motion of the four masses 1.5 kg, 3kg, 2kg and x kg are respectively

$$1.5\,g-T_1=1.5\,\ddot{z}_1 \qquad ..(7)$$

$$3\,g-T_1=3\,\ddot{z}_2 \qquad ..(8)$$

$$2\,g-T_2=2\,\ddot{z}_3 \qquad ..(9)$$

and

$$xg-T_2=x\,\ddot{z}_1\,. \qquad (10)$$

From (4), (5) and (6), we get

$$6g+2T_1-T=0 \quad \Rightarrow \quad T_1=\frac{T-6g}{2} \qquad (11)$$

$$4g+2T_2-T=0 \quad \Rightarrow \quad T_2=\frac{T-4g}{2}\,. \qquad ..(12)$$

Multiplying (7) by 2 and adding with (8), we get

$$6g-3T_1=3(\ddot{z}_1+\ddot{z}_2)=0 \qquad \text{[from (2) and (4)]}$$

Hence, $T_1=2g$.

Hence, from (11), $2g\times 2=T-6g \quad \Rightarrow \quad T=10g$

Therefore, from (12), we get

$$T_2=\frac{10g-4g}{2}=3g.$$

Now, to calculate x, we multiply (9) by x and (10) by 2 and add. We thus obtain

$$4xg-T_2(x+2)=2x\,(\ddot{z}_3+\ddot{z}_4)=0 \qquad \text{[from (3) and (4)]}$$

$$\Rightarrow \qquad 4gx-3g(x+2=0 \qquad [\because \quad T_2=3g]$$

$$\Rightarrow \qquad gx=6g \quad \Rightarrow \quad x=6.$$

Hence, the tension of the string over the fixed pulley is $10g$ N and the value of x is 6.

Ex. 9. A light string passing across a smooth table at right angles to two opposite edges has attached to it at the two ends masses m_1, m_2 $(m_1>m_2)$ which hang vertically. A particle of mass m is attached to the portion of the string lying on the table. Show that the acceleration of the system when left to itself is

$$\frac{m_1-m_2}{m_1+m_2+m}\,g.$$

Since the table is smooth, the acceleration of the whole system of masses will be the same.

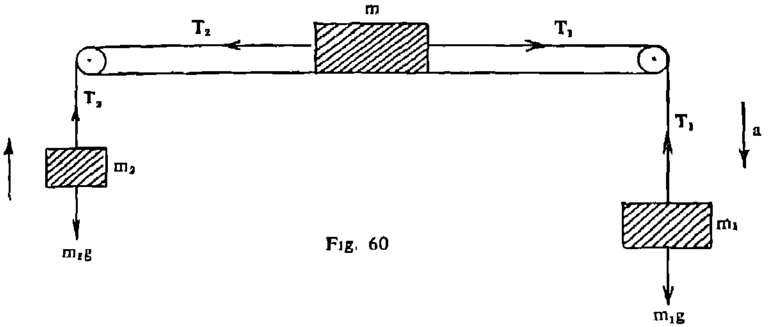


Fig. 60

Let a be the acceleration. Let T_1 be the tension of the string which joins m and m_1 and T_2 be that of the string joining m and m_2.

Now, the equation of motion of the masses m_1, m_2 and m are respectively

$$m_1g-T_1=m_1a \quad \ldots(1)$$

$$T_2-m_2g=m_2a \quad \ldots(2)$$

$$T_1-T_2=ma. \quad \ldots(3)$$

It can be noted here that there is no contribution of the weight of mass m in its horizontal motion, since the component of the vertical weight in the horizontal direction is zero

Adding (1), (2) and (3), we get

$$a(m_1+m_2+m)=(m_1-m_2)\,g$$

$$\Rightarrow \qquad a=\frac{(m_1-m_2)\,g}{m_1+m_2+m}$$

Ex. 10. Two masses m and M are suspended freely over a pulley. If the pulley itself is moved upwards with acceleration f, find the acceleration of each mass and the tension of the string.

Let a be the common acceleration of the masses relative to the pulley and T be the tension of the string. We assume that $M>m$. Then the mass M will move upwards relative to the pulley

From (3) and (5), we get

$$M\ddot{x}=\frac{2\,M\,m\,(g+f)}{M+m}-Mg$$

$$\Rightarrow \qquad \ddot{x}=\frac{2Mm(g+f)}{M+m}-g=\frac{g(m-M)+2mf}{M+m} \qquad (6)$$

From (4) and (5), we get

$$m\,\ddot{y}=\frac{2\,Mm\,(g+f)}{M+m}-mg \quad \Rightarrow \quad \ddot{y}=\frac{2M\,(g+f)}{M+m}-g$$

$$=\frac{(M-m)\,g+2\,Mf}{M+m}$$

Hence, the acceleration of the mass M is

$\frac{(M-m)\,g-2\,mf}{M+m}$ in the downward direction and that of mass m is $\frac{(M-m)+2Mf}{M+m}$ in the upward direction.

Ex. 11. A mass M is drawn up a smooth inclined plane of height h and length l by means of a string passing over the vertex of the plane, from the other end of which hangs a mass m. Show that in order that M may just reach the top of the plane, m must be detached after M has moved through a distance $\frac{M+m}{m}\cdot\frac{hl}{h+l}$.

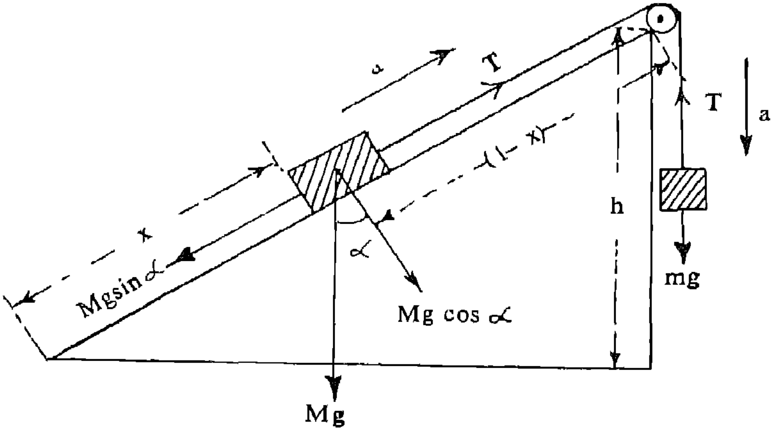


Fig. 62

Let α be the inclination of the plane so that $\sin\alpha=\frac{h}{l}$. If T be the tension of the string, then the equation of motion of masses M and m are respectively

$$T-Mg\sin\alpha=Ma \qquad \ldots(1)$$

and

$$mg-T=ma, \qquad .\ (2)$$

where a is the acceleration of the system.

Adding (1) and (2), we get,

$$a=\frac{g\,(m-M\sin\alpha)}{m+M}=\frac{g\left(m-\frac{Mh}{l}\right)}{m+M}. \qquad ..\ (3)$$

Let us assume that the mass m is detached when the mass M has moved a diatance x over the plane. Let v be the velocity of the mass M just before the mass m is detached. Then using the formula

$v^2=u^2+2\,as$ where $u=0$, we get

$$v^2=2a\,x \qquad \ldots(4)$$

When the mass m is detached, the acceleration of the mass M down the plane is $g\sin\alpha$. This mass, therefore, moves up the plane with retardation $-g\sin\alpha$ till its velocity is zero at the top of the plane i.e. after it describes a distance $l-x$. Hence, using the formula $v^2=u^2-2g\sin\alpha.\,h$, we get

$$0=v^2-2g\sin\alpha.\,(l-x)$$

$$\Rightarrow\quad 2ax=2g\sin\alpha\,(l-x) \qquad [\text{from (4)}]$$

$$\Rightarrow\quad x\,(a+g\sin\alpha)=gl\sin\alpha$$

$$\Rightarrow\quad x=\frac{gl\sin\alpha}{a+g\sin\alpha}=\frac{gl\,.\,\frac{h}{l}}{\frac{g\left(m-\frac{Mh}{l}\right)}{M+m}+\frac{gh}{l}}$$

[from (3)]

$$=\frac{h}{\frac{\left(m-\frac{Mh}{l}\right)l+Mh+mh}{(M+m).l}}$$

$$=\frac{h\,(M+m)\,l}{m\,(h+l)}=\frac{M+m}{m}\cdot\frac{hl}{h+l}.$$

Ex. 12. Two bodies of masses 4 kg and 2 kg rest on two smooth inclined planes of elevation 30° and 60° respectively, being connected by a string passing over a pulley at the common vertex of the planes. Find the distance they travel in 2s from rest.

The component of weight $4g$ N down the plane is $4g \sin 30°\text{N}=2\,g$ N and that of $2g$ N down the plane is $2g \sin 60°$ N

$$=2g\sqrt{\frac{3}{2}}\ \text{N}=\sqrt{3}\ g\ \text{N}$$

which is less than the previous component. Hence, the 4 kg mass will move down the inclined plane and the 2 kg mass will move up the other inclined plane. Let a m/s² be the common acceleration of the system and TN be the tension of the string, then the equations of motion of the masses on the inclined planes are

$$4g \sin 30°-T=4a$$

$$\Rightarrow \quad 2g-T=3a \qquad (1)$$

$$\text{and} \quad T-2g \sin 60°=2a$$

$$\Rightarrow \quad T-\sqrt{3}\ g=2a \qquad (2)$$

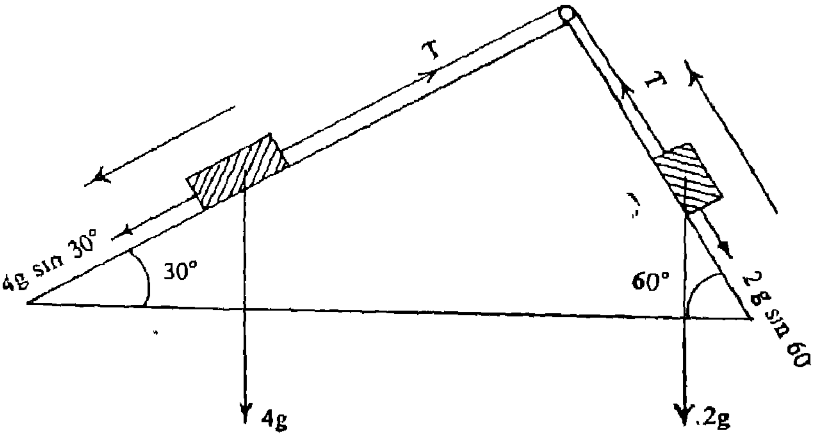


Fig. 63

Adding (1) and (2), we get

$$(2-\sqrt{3})\,g=5a$$

$$a=\frac{(2-\sqrt{3})\,g}{5}.$$

Now, using the formula $s=ut+\frac{1}{2}at^2$, we get the required distance travelled by the bodies in 2s with the above acceleration. Hence, the required distance

$$=\frac{1}{2}at^2=\frac{1}{2}\times\frac{(2-\sqrt{3})g}{5}\times 4\text{m}=4(2-\sqrt{3})\text{ m},$$

taking $g=10$ m/s^2.

EXERCISE ON CHAPTER 7

1. Two masses, 5 kg and 6 kg, are suspended by an inextensible string passing over a frictionless pulley. Find the tension of the string and the acceleration of the system. [Ans. 53.5 N; .9 m/s^2]
2. Two masses, 3kg and 7 kg, are connected by a light string passing over a smooth pulley, and hangs freely Motion is allowed to start from rest In what time will the heavier mass descend 8 m ? Find also the distance described during the next $1\frac{1}{2}$ s. (Take $g=10$ m/s^2) [Ans 2s ; 16.5 m]
3. Two particles are connected by a light inextensible string passing over a smooth fixed pulley. If the system moves with an acceleration of 4.9 m/s^2, compare the masses of the particles. [Ans 3 : 1]
4. Two scale-pans of mass .1 kg each, hang freely from the two ends of a string passing over a smooth pulley On these are placed two masses 37 kg and .41 kg respectively, and the system is allowed to start with the pans in the same horizental level. Find the velocity of the system when the vertical distance between the pans is .4 m. [Ans .4 m/s]
5. Two masses, each equal to 3 kg, are connected by a string and hang over a pulley. A mass of 2 kg is now added to one of them. Find by how much the pressure on the pulley is increased [Ans. 14.7 N]
6. A light inextensible string passes over a smooth fixed pulley. If one of the masses suspended from it be 3 kg and the

acceleration of the masses be 4.9 m/s^2, show that the other mass is either 9 kg or 1 kg

7 .26 kg and m kg ($m < .26$) masses are connected by a light inextensible string Each load moves through 2 7 m in the first 3 s after the commencement of motion What is the value of m and acceleration ? [Ans .23 kg, .6 m/s^2]

8. Two particles of masses .01 kg and 02 kg are connected by a light inextensible string passing over a smooth fixed pulley. The smaller mass moves on a smooth horizontal plane while the larger vertically. If the space described in 1 s is 3.27 m, find g. [Ans. 9.81 m/s^2]

9. Two bodies of masses 5 175 kg and 2 175 kg are connected by a light string passing over a smooth pulley Find

(i) the velocity at the end of 4s.

(ii) the space described in 4s from the commencement of motion.

If the string is snapped at the end of 4 s, find the distance described by each particle in the next 6 s.

[Ans. (i) 16 m/s (ii) 32 m, 272 4 m]

10 A smooth inclined plane whose height is one-half of its length has a small smooth pulley at the top, over which a string passes To one end of the string is attached a mass of 11.25 kg which rests on the plane, while from the other end which hangs vertically, is suspended a mass of 8.35 kg, and the masses are free to move. Find the acceleration, and the distance traversed by either mass in 2s. Find also the pressure on the pulley.

[Ans, 1.3625 m/s^2, 2.725 m, 122.03 N]

11. A mass of 3 kg decending vertically draws up a mass of 2 kg by means of a light string passing over a pulley. At the end of 5s the string breaks. Find how much higher the 2kg mass will go before it starts descending

[Ans. 4.9m after the string breaks]

12. Two masses m_1, m_2 ($m_1 > m_2$) are suspended by a light inextensible string passing over a smooth, fixed pulley. If the tension in the string be equal to the weight of mass M, prove that M is the harmonic mean between m_1 and m_2.

13. Masses 3.5 kg and 6.3 kg are connected by a string passing over a smooth pulley and motion starts from rest. At the end of 5s, a portion of mass 4.9 kg breaks off the larger mass. How

much further do the masses go before they first come to stop and when does this happen ? If the masses are to stop in 5 s after the incident, what should be the mass of the portion which breaks off ?

$$\left[\text{Ans. } \frac{70}{3}\,m, \frac{10}{3}\,\text{s}, \ \frac{196}{45}\,\text{kg}\right]$$

14 Two masses 10 kg and 3 kg respectively are connected by a fine string which passes over a smooth pulley fixed at the head of a smooth inclined plane, 5 m long and 1 m high The heavier particle is on the plane and the lighter one just hangs over the pulley, the string being 5 *m* long. Find the acceleration of the masses and the tension of the string How long will it be after the 4 kg mass reaches the ground before the string is again taut ? [Ans. 1.4 m/s^2, 33 6 N, 1.7s nearly]

15 Two masses each equal to m kg are connected by a light string passing over a smooth pulley. What mass must be taken from one and added to the other so that the system may describe 90 m in 15 s ? $\left[\text{Ans } \frac{4m}{49}\text{kg}\right]$

16. Two unequal masses, connected by a string, hang over a pulley. If the sum of the masses be constant, show that the greater the acceleration, the less is the tension in the string [C.U. '53]

17. A flexible heavy chain of length $2\,l$ is moving over a smooth fixed pulley, the two unequal portions of it hanging vertically Prove that at the instant when its middle point is at a distance x below the pulley. the acceleration with which it is moving is $\frac{x}{l}\,g$.

18. A mass of 5 kg descending vertically draws up a mass of 3 kg by means of a light string passing over a smooth pulley At the end of 8 s, the string breaks. Find how much higher the 3 kg mass will go. [Ans. 19.6 m]

19. Two bodies of masses 1 kg and 2 kg are connected by a light inextensible string passing over a smooth fixed pulley. Initially, the 1 kg mass is 1 m below the 2 kg mass. How long after the weight starts to move will their centres of gravity be at the same height ? [Ans. .55 s nearly]

20. A mass of .78 kg lying on a smooth horizontal table 1 m from the edge is drawn along the table by a mass of .2 kg hanging

freely by means of a light inextensible string passing over a smooth pulley at the edge. How long does it take to reach the edge, and what is the pressure on the pulley ?

$$\left[\text{Ans. } 1\text{ s};\ \frac{39\sqrt{2}}{25}\text{ N}\right]$$

21. A mass of .5 kg is placed on a smooth horizontal platform 4 m high at a distance of 20 m from the edge and connected by a light string, 20 m long, with a mass of .3 kg on the edge of the platform. If 3 kg mass is pushed gently over the edge, find (i) how long it takes to reach the ground, and (ii) how much longer the 5 kg mass takes to reach the edge.

[Ans. (i) 1.475 s (ii) 2.95 s more]

22 A string, hung over a pulley, has at one end a mass of 10 kg and at the other end masses of 4 kg and 8 kg respectively. After being in motion for 5 s, the 4 kg mass is taken off Find how much further the masses go before they first come to instantaneous rest. [Ans. 9.11 m nearly]

23 If two unequal weights be contained in scale pans connected by a string passing over a smooth pulley, prove that if the weights of the pans be negligible, the pressure between each pan and the contained weight is equal to the tension of the string.

24. A mass of 4 kg is attached by a string passing over a smooth pulley to a larger mass Find the magnitude of the latter so that if after the motion has continued for 3 s, the string is cut, the former will ascend 5 m before descending.

[Ans .495 kg nearly]

25. Two masses m_1 and m_2 $(m_1 > m_2)$ connected by a string hangs over a pulley After 1 s from start, m_1 is suddenly stopped and instantly let go. Show that the time that elapses before the string becomes tight again is

$$\frac{m_1 - m_2}{m_1 + m_2} \text{ seconds.}$$

26 Two equal masses hang at rest over a smooth pulley. One is projected upwards with a velocity of 19.6 m/s. In what time will the string become tight again ? [Ans 2s]

27. Two strings pass over a smooth pulley. On one side, masses 3 kg and 5 kg are attached to the ends of the strings and on the other side, a single mass of 6 kg is fastened to the other

ends. Find the tensions of the strings and the acceleration of the system. (Portions of the strings on both sides of the pulley are assumed to be vertical). [Ans. 42 N, 25 2 N, 1.4 m/s^2]

28. A light string carrying to unequal weights and passing over a smooth pulley can only just bear a tension equal to $\frac{12}{25}$ of the sum of the weights. Prove that the least acceleratian possible of the system is $\frac{1}{5}$ g, and that the lighter mass cannot exceed $\frac{2}{5}$th of the total mass

29. Two equal scale-pans are suspended over a pulley, the mass of each being 6 kg. How a mass of 8 kg is to be divided between the scale-pans so that the heavier mass may describe .98 m in the third second of motion ? [Ans 4.4 kg and 3.6 kg]

30. Masses of .4 kg and .6 kg hang over a pulley. The 4 kg mass is projected downwards with a velocity of 3 92 m/s When does it first come to rest ? What distance is described by each mass in this time ? When will each mass come back to its original position ? [Ans 2 s; 3 92m; after 2 s more]

31. Two equal masses of 3 kg each are connected by a light string hanging over a smooth pulley If the string breaks under a tension of 4 N, find how great a mass may be put on one of the 3 kg masses before this catastrophe occurs.

[Ans. 3 kg]

32. A pulley carrying a total load W hangs in a loop of cord which passes over two fixed pulleys and has unequal weights P and Q freely suspended from its ends, each segment of the cord being vertical. Show that W will remain at rest provided

$$\frac{1}{P}+\frac{1}{Q}=\frac{4}{W}.$$

[C.U. '39, '42]

33 Two identical weights, each of mass M, are connected by a light inextensible string passing over a smooth fixed pulley rotating about a horizontal axle. A small weight of mass m is placed on one of the weights. Show that the force with which it will press M is

$$\frac{2mM}{2M+m}g.$$

34. Two masses of 1 kg and .2 kg are connected by a weightless string passing over a fixed, smooth pulley. Initially, both the weights are 2m above the floor. Find (i) the tension in the string and (ii) the time required by .2 kg mass to reach the floor

[Ans (i) 1.3067 N nearly (ii) 1 10 s nearly]

35. A pulley carrying a load W hangs in a loop of a cord, one end of which is fixed at a point and the other end passes round a fixed pulley, each segment of the cord being vertical. What force F should be applied to the end of the string passing round the fixed pulley for the weight W to (i) remain at rest, (ii) move upwards with an accelreation a or (iii) move downwards with an acceleration a ? Neglect masses of the pulleys and the strings.

$$\left[\text{Ans. (i) } F=\frac{W}{2} \text{ (ii) } F=\frac{W}{2}\left(1+\frac{a}{g}\right)\right.$$
$$\left.\text{(iii) } F=\frac{W}{2}\left(1-\frac{a}{g}\right)\right]$$

36. A light string passes over a light fixed pulley. It carries a mass P at one extremity and a light pulley at the other. Another light string passes over this second pulley, carrying masses R and Q at its extremities. The system starts from rest. Find the acceleration of the masses. Show that if R always remains at rest, then

$$\frac{4}{P}+\frac{1}{Q}=\frac{3}{R}$$

and that if P always remains at rest, then

$$\frac{4}{P}=\frac{1}{Q}+\frac{1}{R}.$$

$$\left[\text{Ans. Acceleration of } P=\frac{g\left(\frac{1}{Q}+\frac{1}{R}-\frac{4}{P}\right)}{\frac{1}{Q}+\frac{1}{R}+\frac{4}{P}},\right.$$

acceleration of $Q=g-\frac{T'}{Q}$ and that of $R=g-\frac{T'}{R}$, where

$$\left.T'=\frac{4g}{\frac{1}{Q}+\frac{1}{R}+\frac{4}{P}}\right]$$

37. Two weights W and W' are connected by a light string passing over a smooth pulley. If the pulley moves vertically upwards

with an acceleration equal to that of gravity, show that the tension of the string is

$$\frac{4W\,W'}{W+W'}.$$ [C.U. '44]

38. One end of a string is fixed. It then passes under a movable pulley to which a weight W is attached. The string then passes over a fixed pulley, and a weight P is attached to its other end, all the three sections of the string being vertical. Show that, neglecting the masses of the pulleys, the acceleration with which W ascends is

$$\frac{2P-W}{W+4P}\,g.$$

Find also the tension of the string. [C.U. '73]

$\left[\text{Ans. } \dfrac{3\,WP}{W+4P}\right]$

39. A light string passes over one fixed peg under a movable pulley, and then over a second fixed peg. A mass m_1 is attached to one end of the string, a mass m_2 to the other end and a mass M to the movable pulley. Assuming the pulley to be light and the portion of the string, not round the pulley, to be vertical, prove that the tension of the string is

$$\frac{4\,M\,m_1\,m_2\,g}{M\,(m_1+m_2)+4m_1m_2}.$$

40. Two pulleys, each of mass m, are connected by a string hanging over a smooth fixed pulley. A string with masses $2m$ and $3m$ at its ends is hung over one pulley and another with masses m and $4m$ over the other If the system is free to move, show that the accleration of either pulley is $\dfrac{4\,g}{25}$.

41. A small pulley carrying a total load W hangs in a loop of a cord which passes over two fixed pulleys, and has unequal weights, P and Q, freely suspended from its ends, each segment of the cord being vertical. Show that W will ascend with acceleration

$$\frac{4\,PQ-W\,(P+Q)}{4\,PQ+W\,(P+Q)}\,g.$$

42. A smooth and stationary pulley is secured to the end of a light bar. The bar is secured in a vertical direction on a pan of a

balance. Two masses of 3 kg and 1 kg are attached to the ends of the string passing over the pulley on the vertical bar without friction. Determine (i) the force with which the pulley acts on the bar and (ii) the reading of the balance, when the weights move. (iii) What will be the case if the masses interchange the ends of the string ? [Ans (i) 29.4 N (ii) 3 kg (iii) There will be no change in the reading of the balance]

43. Two monkeys of the same weights are hanging at the ends of a light rope passing over a smooth pulley Initially, both the monkeys are at the same height above the floor. One monkey begins to climb the rope while the other stays where it is relative to the rope. Where will the second monkey be when the first one reaches the pulley?

[Ans Both the monkeys will reach the pulley together]

44. A rope hangs down over a smooth pulley and a man of 84 kg moves down the protion of the rope on one side of the pulley with an acceleration of ·5 m/s^2. With what uniform acceleration, another man of 78 kg must move up the other portion of the rope so that the rope may remain at rest. [Ans. ·215 m/s^2 nearly]

45. On one side of a light string passing over a smooth pulley, a weight W hangs. A boy takes hold of the other end and begins to climb up the rope with uniform acceleration rising 4m in 2s. If W remains at rest, find the weight of the boy.

$$\left[\text{Ans. } \frac{49}{59}\,W\right]$$

46. A mass of 2 m kg lying on a smooth horizontal table is attached to a light inextensible string which passes over the smooth edge of the table, and carries a smooth light pulley over which hangs a similar string carrying masses $(m+x)$ kg and $(m-x)$ kg at its ends. Prove that, after the system is released, the acceleration of the mass on the table is

$$\frac{m^2-x^2}{2m^2-x^2}\,g.$$

47. Two masses $m_1 = .2$ kg and $m_2 = .3$ kg are connected by a thread and are placed on a smooth horizontal surface of a table. A force $F = 1$ N is applied to the mass m_1, parallel to the surface of the table. (i) Determine the tension in the thread linking the two masses. (ii) If the thread can bear a maximum load of mass 1 kg, determine the maximum value of F if it is applied to (a) the mass m_1 (b) the mass m_2.

(Take $g=10$ m/s^2) [Ans. (i) .6N (ii) (a) 16.67N (b) 25N]

48. Four identical blocks each of mass m are connected by similar strings in a row and placed on a smooth horizontal table top. A force F is applied to the first block. Find the tensions in the strings.

$$\left[\text{Ans. } \frac{3}{4}F, \frac{1}{2}F, \frac{1}{4}F\right]$$

49. A mass .5 kg of a body A is placed on a smooth table with a string attached to it. The string passes over a frictionless pulley and is connected to another mass .2 kg of a body B. Initially, the mass of A is at a distance of 2 m from the edge of the table and moving with a speed of .5 m/s away from the pulley. What will be the (i) position and (ii) velocity after 1 second ?

[I.I.T. '75]

[Ans (i) .9 m towards the pulley from the initial position
(ii) 2.3 m/s towards the pulley]

50. Two bodies P and Q having masses 9 kg and 6 kg respectively are connected by a string passing over the top of an inclined plane of inclination 30° to the horizon. One body rests on the plane and the other hangs vertically. Show that P hanging vertically will drag Q up the whole length of the plane in half the time that Q, hanging vertically, will take to drag P up the plane.

51. Two smooth inclined planes of equal heights, whsoe inclinations to the horizon are 30° and 60°, are placed back to back. Two bodies of masses 6 kg and 10 kg placed on them respectively are connected by a light inextensible string passing over a smooth pulley at the common vertex of the planes. Find the tension in the string and the accleration of the system.

$$\left[\text{Ans. } \frac{147\,(1+\sqrt{3})}{8}\ N;\ \frac{40\,(5\sqrt{3}-3)\text{m/s}^2}{80}\right]$$

Chapter 8

Projectiles

8.1. Introduction

In the previous chapters, we have considered various kinds of motion of a particle or a body viz. motion in a straight line, vertical motion, motion on an inclined plane under gravity and motion of a stytem of bodies connected by a string. In this chapter, we shall consider the free motion of a particle when it is projected in any direction in space While considering this type of motion, we shall consider the following assumptions for simplicity : (i) the motion of the particle takes place in a vertical plane, (ii) the acceleration due to gravity, g, is uniform and acts in the same vertical direction at all points of the path of the particle and (iii) there is no resistance due to air during the motion of the particle. When a particle is projected upwards with a velocity at an angle α with the horizon, the vertical component of the velocity will continually change whereas the horizontal component of the velocy will remain unchanged since the component of the acceleration, g, in the horizontal direction is zero. Hence, the resultant direction of motion will continually change and the particle will describe a curved path. It will be shown later that this path is a parabola.

8.2. Some definitions

A body projected in any direction in space is called a *projectile* and the curved path described by a projectile is called its *trajectory*. The initial velocity with which a particle is projected is in space is called the *velocity of projection*. The angle which the direction of projection makes with the horizon is called the angle of projection. The distance measured between the point of projection and the point where the particle strikes a given plane through the point of projection is called its *range* on the plane. When the given plane is horizontal, we call it the *horizontal range*. The time interval between the instant of projection and the instant when the particle meets a fixed plane through the point of projection i.e. the time for which the projectile remains in air is called the *time of flight*.

8.3. Position and Velocity of a Particle after any Time

Let a particle be projected from O with a velocity u in a direction making an angle α with the horizon OA. Let v be the velocity of the particle at P after any time t. From P, we draw $PM \perp OA$. We choose the line OA as the x-axis and a line through O perpendicular to OA as the y-axis. Let $OM=x$ and $MP=y$ so that the co-ordinates of P are (x, y). Now, the horizontal and the vertical components of u are respectively $u \cos \alpha$ and $u \sin \alpha$ Let the direction of v make an angle θ with the horizontal. Hence, the horizontal and the vertical components of v are $v \cos \theta$ and $v \sin \theta$ respectively. Sine in the horizontal direction, the particle moves with a constant velocity, we have

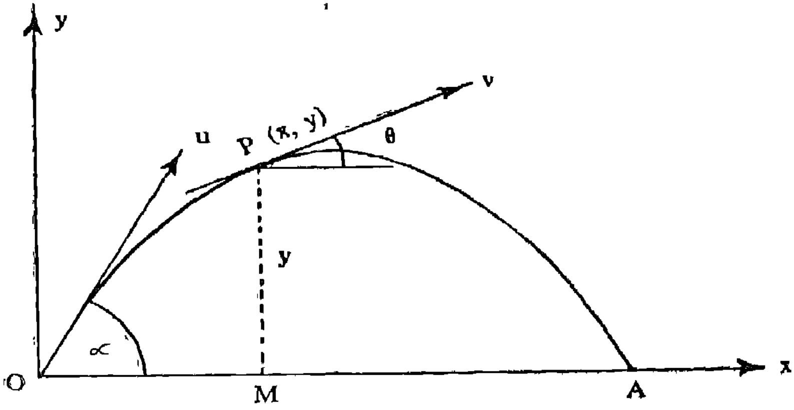


Fig. 64

$$v \cos \theta = u \cos \alpha \qquad \ldots(1)$$

and
$$x = ut \cos \alpha. \qquad \ldots(2)$$

Considering the vertical upward motion of the particle from O to P, and using the formula $v=u-gt$ and $h=ut-\frac{1}{2}gt^2$,

we get

$$v \sin \theta = u \sin \alpha - gt \qquad \ldots(3)$$

and
$$y = ut \sin \alpha - \frac{1}{2}gt^2. \qquad \ldots(2)$$

From (1) and (3), squaring and adding, we get

$$v^2=u^2-2\,ugt\ \sin\alpha+g^2t^2. \qquad (5)$$

Dividing (3) by (1), we get

$$\tan\theta=\frac{u\sin\alpha-gt}{u\cos\alpha}\ . \qquad (6)$$

Equations (5) and (6) give the magnitude and direction of the velocity after time t and equations (2) and (4) give the position of the particle after time t.

8 4. The Equation of Trajectory

Let a particle be projected from O with a velocity u at an angle α with the horizon. Let the horizontal and vertical through O in the plane of projection be chosen as x and y axes respectively. Let P be the position of the particle at any time t and let (x, y) be the co-ordinates of P.

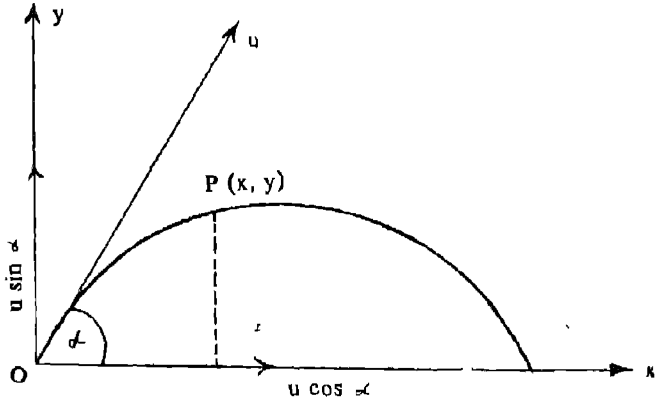


Fig 65

The horizontal and vertical components of u are u cos α and u sin α respectively.

Considering the horizontal motion of the particle, we have

$$x=u\cos\alpha.\ t \qquad \text{...(1)}$$

Considering the vertical motion of the particle, we have

$$y=u\sin\alpha.\ t-\frac{1}{2}\,gt^2 \qquad \text{...(2)}$$

Eliminating t from (1) and (2) by substituting the value of t from (1) in (2), we get

$$y=u\sin\alpha.\ \frac{x}{u\cos\alpha}-\frac{g}{2}\ .\ \frac{x^2}{u^2\cos^2\alpha}$$

$$=x\tan\alpha-\frac{g}{2u^2\cos^2\alpha}x^2, \qquad \text{.. (3)}$$

which is of the form

$$y=Ax+Bx^2.$$

But this is a general equation of a parabola. Hence, (3) represents a parabola.

The equation (3) can be written as

$$x^2-\frac{2u^2\sin\alpha\cos\alpha}{g}x=-\frac{2u^2\cos^2\alpha}{g}y$$

$$\Rightarrow\quad\left(x-\frac{u^2\sin\alpha\ \text{cas}\ \alpha}{g}\right)^2=-\frac{2u^2\cos^2\alpha}{g}\left(y-\frac{u^2\sin^2\alpha}{2g}\right),$$

which shows that the vextex of the parabola is the point $\left(\frac{u^2\sin\alpha\cos\alpha}{g},\ \frac{u^2\sin^2\alpha}{2g}\right)$ and the latus rectum is $\frac{2u^2\cos^2\alpha}{g}$, since the co-efficient of y is negative, the axis of the parabola is on the vertically downward direction and the co-ordinates of the vertex are $\left(\frac{u^2\sin 2\alpha}{2g},\ \frac{u^2\sin^2\alpha}{2g}\right)$ and those of the focus are $\left(\frac{u^2\sin 2\alpha}{2g},\ -\frac{u^2\cos 2\alpha}{2g}\right)$.

8.5. Velocity of a Particle at any Height h

In the figure of section 8.3, if h be the height of the particle at P above the horizon and v be its velocity there, whose direction makes an angle θ with the horizontal, then considering the horizontal motion, we have

$$v\cos\theta=u\cos\alpha. \qquad \text{...(1)}$$

Considering the vertical motion of the particle from O to P and using the formula

$$v^2=u^2-2gh, \text{ we get}$$

$$v^2\sin^2\theta=u^2\sin^2\alpha-2gh. \qquad \text{...(2)}$$

Squaring (1) and adding with (2), we get

$$v^2=u^2-2gh \qquad (3)$$

From (2), $v \sin \theta=\sqrt{u^2 \sin^2\alpha -2gh}$. (4)

Dividing (4) by (1), we get

$$\tan \theta=\frac{\sqrt{u^2 \sin^2\alpha -2gh}}{u \cos \alpha}. \qquad (5)$$

The equations (3) and (5) give the magnitude and direction of the velocity at a height h.

8 6. Greatest Height attainee by a Projectile and Time to greatest Height

Let u be the velocity of projection and α be the angle of projection, O being the point of projection. Let $AM=H$ be the greatest height and T_1 be the time to reach the greatest height At the greatest height A, the vertical component of the velocity is zero. Considering the vertical motion of the projectile and using the formula $v^2=u^2-2gh$, we get

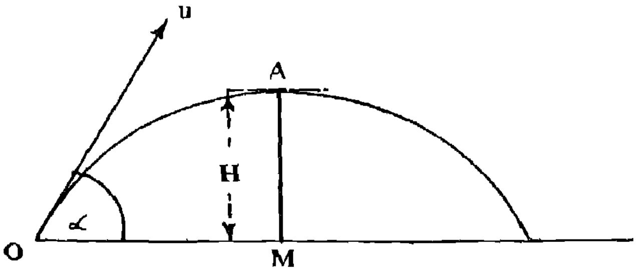


Fig 66

$$0=u^2 \sin^2\alpha-2g\ H$$

$$\Rightarrow \qquad H=\frac{u^2 \sin^2\alpha}{2g}. \qquad (1)$$

To find the time T_1 to attain the greatest height, we use the formula

$v=u-gt$ and get

$$0=u \sin \alpha -gT_1$$

— $u \sin \alpha$

8 7. Horizontal Range and Time of Flight of a Projectile

Let u and α be respectively the velocity and angle of projection of a particle, O being the point of projection. Let $R=OA$ be the range on the horizontal plane through O and T, the time of flight. When the particle returns to the horizontal plane after time t, the vertical displacement is zero. Hence, considering the vertical motion and using the formula

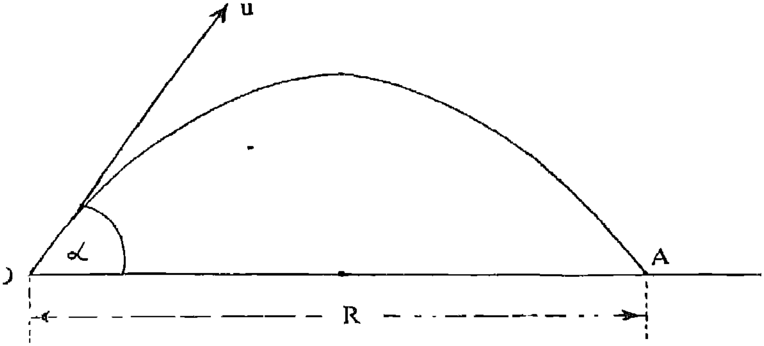


Fig. 67

$$h=ut-\frac{1}{2}gt^2, \text{ we get}$$

$$0=uT\sin\alpha-\frac{1}{2}g\ T^2$$

$$\Rightarrow \qquad T=\frac{2u\sin\alpha}{g}. \qquad (1)$$

In the horizontal direction, there is no acceleration and hence the particle moves with the constant horizontal velocity $u\cos\alpha$. Since the particle reaches A after time T,

$$OA=R=u\cos\alpha\ .\ T=u\cos\alpha\ .\ \frac{2\ u\sin\alpha}{g}$$

$$\Rightarrow \qquad R=\frac{u^2}{g}\sin 2\alpha \qquad ...(2)$$

Hence, the time of flight and the horizontal range are given by the equations (1) and (2).

Maximum horizontal range and its corresponding angle of projection :

We see from (2) that for a given value of u, R is maximum when

$$\sin 2\alpha=1 \text{ i.e } 2\alpha=90^\circ \text{ or } \alpha=45^\circ.$$

Hence, *the horizontal range is maximum when the angle of projection is 45° and the maximum horizontal range is* $\frac{u^2}{g}$. In this case we see that when the horizontal range is maximum, the direction of projection bisects the angle between the horizontal and the vertical.

Two directions of projection for a given range :

We have seen that if the angle of projection is α, then the horizontal range $R=\frac{u^2}{g}\sin 2\alpha$.

Now, since $\sin(180^\circ-2\alpha)=\sin 2\alpha$, hence we get the same value of the range R when the angle of projection is $\frac{190^\circ-2\alpha}{2}=90^\circ-\alpha$ Hence, there are two directions $\alpha_1=\alpha$ and $\alpha_2=90^\circ-\alpha$ of projection for which the horizontal range remains the same. Now, the angle of projection for maximum range is 45°

We observe that

$$45^\circ-\alpha_1=\alpha_2-45^\circ$$

Hence, *for a given horizontal range with the same velocity of projection, there are two directions of projection which are equally inclined to the direction of the maximum range.*

8.8. Range on an inclined Plane

Let a particle be projected with a velocity u at an angle α with the horizon from a point O. Let OA be an inclined plane through O, of inclination β to the horizon. Let the particle move in the vertical plane through the line of greatest slope of the inclined plane.

Let A be the point where the projectile meets the inclined plane after describing the path Let $R=OA$ be the range on the inclined plane and T be the time of flight on the inclined plane

We shall now consider the motion of the particle along and perpendicular to the plane. The component of u perpendicular to the plane is $u\sin(\alpha-\beta)$ and that of g in the same direction is $-g\cos\beta$. The projectile lies on the same inclined plane after time T. Hence, the displacement of the projectile perpendicular to the plane is zero.

Considering the motion perpendicular to the plane and using the formula $h=ut-\frac{1}{2}gt^2$, we get

$$0=u\sin(\alpha-\beta)\,.\,T-\frac{1}{2}g\cos\beta\ \ T^2$$

$$\Rightarrow\qquad T=\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\,. \tag{1}$$

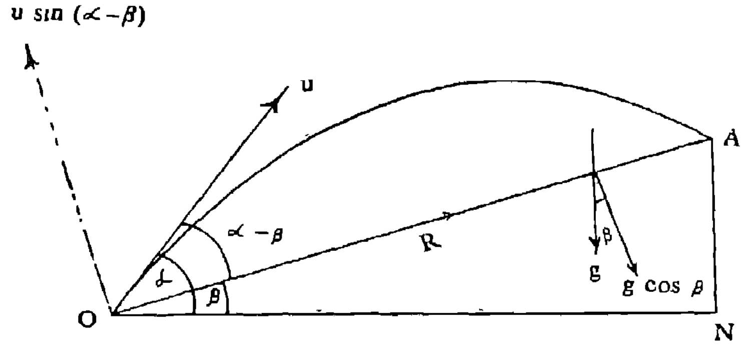


Fig 68

The component of velocity in the horizontal direction is constant and is equal to $u\cos\alpha$ throughout the motion.

Let AN be perpendicular to the horizontal plane.

$$\therefore\qquad ON=u\cos\alpha\,.\,T$$

$$\Rightarrow\qquad R\cos\beta=u\cos\alpha\,.\,T$$

$$\Rightarrow\qquad R=\frac{u\cos\alpha}{\cos\beta}\times\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\qquad\text{[from (1)}$$

$$\Rightarrow\qquad R=\frac{u^2}{g\cos^2\beta}\Big[\sin(2\alpha-\beta)-\sin\beta\Big]. \tag{2}$$

For a given u and β, R is maximum when $\sin(2\alpha-\beta)=1$

$$\therefore\qquad R_{\max}=\frac{u^2}{\cos^2\beta}(1-\sin\beta)=\frac{u^2}{g(1+\sin\beta)}\,.$$

Now, when $\sin(2\alpha-\beta)=1$, then $2\alpha-\beta=\frac{\pi}{2}\Rightarrow\alpha=\frac{1}{2}\left(\frac{\pi}{2}+\beta\right)$.

Thus the range on the inclined plane will be maximum when the direction of projection bisects the angle between the horizontal and the normal to the plane i e. the angle between the plane and the vertical.

8.9. Solved Examples

Ex. 1 A particle is projected at an angle $\sin^{-1}\dfrac{3}{5}$ to the horizon with a velocity of 25 m/s. Find its position and its velocity at the end of 2 s. When will it be at a height of 5 m and what will be its velocity then ? [Take $g=10$ m/s]

Let α be the angle of projection. Then $\sin\alpha=\dfrac{3}{5}$.

$$\cos\alpha=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$$

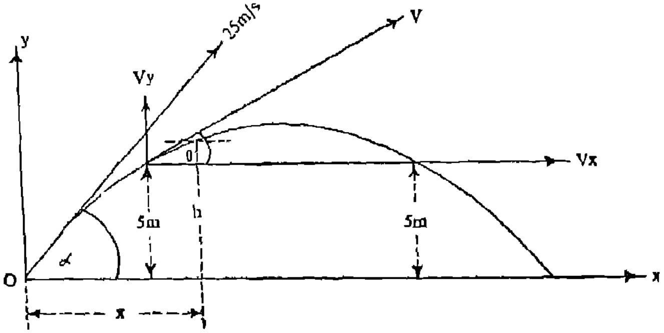


Fig 69

Now, the vertical component of the velocity is $u\sin\alpha=25\times\dfrac{3}{5}$ m/s$=15$ m/s

and the horizontal component of the velocity is $u\cos\alpha=25\times\dfrac{4}{5}$ m/s $=20$ m/s.

If hm be the height of the particle after 2 s, then considering the vertical motion of the particle and using the formula

$$h=u\sin\alpha\ t-\frac{1}{2}gt^2,\ \text{we get}$$

$$h=15\times2-\frac{1}{2}\times10\times4=30-20=10$$

Hence, the particle will be at a height of 10 m after 2 s.

If xm be the horizontal distance of the particle after 2 s, then we have

$$x=\text{horizontal velocity}\times\text{time}$$

$$=20\times2\text{m}=40\text{m}.$$

Hence, the horizontal distance of the particle is 40 m

Let after t s, the particle be at a height of 5 m and let v m/s be its velocity there. Then using the formulas

$$h=u\sin\alpha\, t-\frac{1}{2}gt^2 \text{ and } v=u-gt, \text{ we get}$$

$$5=15\,t-5t^2$$

$$\Rightarrow\quad t^2-3t+1=0$$

$$\Rightarrow\quad t=\frac{3\pm\sqrt{9-4}}{2}=\frac{3\pm\sqrt{5}}{2}$$

$$\simeq\frac{3\pm2.24}{2}$$

$$=2.62 \text{ or } .38.$$

Hence, the particle will be at the height of 5 m twice after .38 s and 2.62 s nearly.

If v_x m/s and v_y m/s be the horizontal and vertical components of the velocity of the particle at a height of 5 m, then considering the vertical motion and using the formula

$$v^2=u^2\sin^2\alpha-2gh, \text{ we get}$$

$$v^2_y=15^2-2\times10\times5=225-100=125$$

$$\therefore\quad v_y=5\sqrt{5}.$$

Also, since in the horizontal direction there is no acceleration, the particle moves with constant velocity viz. 20 m/s in the horizontal direction.

Hence, the resultant velocity of the particle at a height of 5 m is

$$\sqrt{(5\sqrt{5})^2+20^2}\text{ m/s}=\sqrt{125+400}\text{ m/s}$$

$$=\sqrt{525}\text{ m/s}=5\sqrt{21}\text{ m/s}$$

If this resultant velocity makes an angle θ with the horizontal, then

$$\tan\theta=\frac{5\sqrt{5}}{20}=\frac{\sqrt{5}}{4}$$

so that $$\theta=\tan^{-1}\frac{\sqrt{5}}{4}.$$

Ex 2. A cricket ball struck from the ground pitches 100 m ahead after rising to a maximum height of 20 m. Find the time of flight and the direction in which it is struck.

Let u m/s be the velocity of projection and α be the angle of projection.

The horizontal range $=\dfrac{u^2}{g}\sin 2\alpha=100.$ $\quad$ ..(1)

Also the greatest height $=\dfrac{u^2\sin^2\alpha}{2g}=20.$ $\quad$ (2)

Dividing (1) by (2), we get

$$\tan\alpha=\frac{4}{5}$$

$$\therefore \qquad \sin\alpha=\frac{4}{\sqrt{41}}.$$

Hence, from (2), we have

$$u^2=\frac{40\,g}{\sin^2\alpha}=\frac{40\times 9.8\times 41}{16}=\frac{20009}{2}$$

$$\Rightarrow \qquad u=\sqrt{\frac{2009}{2}}=\frac{\sqrt{4018}}{2}$$

Hence, the velocity of projection is $\dfrac{\sqrt{4018}}{2}$ m/s

Hence, the time of flight $=\dfrac{2u\sin\alpha}{g}$

$$=\sqrt{4018}\times\frac{4}{\sqrt{41}}\times\frac{1}{9.8}\text{ s}$$

$$=\frac{20\sqrt{7}}{7}\text{ s.}$$

Ex. 3. A stone is dropped from a balloon moving horizontally with a velocity of 30 m/s and reaches the ground in 4s. Find the height of the balloon and the velocity of the stone on striking the ground.

[Take $g=10$ m/s]

Let h m be the height of the balloon. Since the initial velocity of the stone is in the horizontal direction, its vertical component is zero. Considering the vertical motion of the stone, we get

$$h=\frac{1}{2}\times 10\times 4^2$$
$$=80.$$

Let v_x m/s and v_y m/s be respectively the horizontal and vertical components of the velocity of the stone on striking the ground.

Then considering the vertical motion, we get

$$v_y^2=2\times 10\times 80=1600$$

$$\Rightarrow \qquad v_y=\sqrt{1600}=40.$$

Also, V_x= initial horizontal velocity=30 m/s

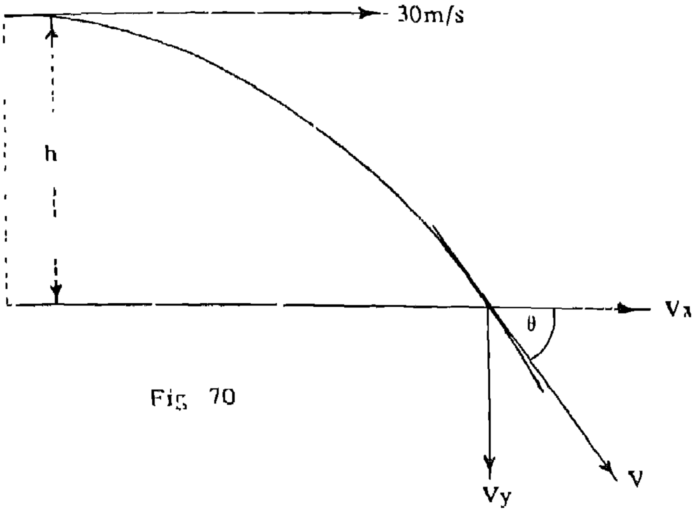


Fig. 70

If V m/s be the resultant velocity of the stone on striking the ground and if it makes an anlge θ with the horizon, then

$$V=\sqrt{V_x^2+V_y^2}$$
$$=\sqrt{900+1600}=50$$
$$\text{and } \tan\theta=\frac{40}{30}=\frac{4}{3}.$$

Hence, the velocity of the stone on striking the ground is 50 m/s making an angle $\tan^{-1}\frac{4}{3}$ with the horizon.

Ex. 4. A particle is projected so as to pass through two points whose horizontal distances from the point of projection are 12 m and 24m, and which are at vertical heights of 4m and 5m above the horizontal plane through the point of projection. Find the velocity and the direction of projection.

Let u m/s and α be the velocity and the angle of projection respectively. Let t s be the time to reach the first point. Considering the horizontal and verticle motions of the particle separately in this case, we get

$$12=u\cos\alpha \,.\, t \qquad \ldots(1)$$

and

$$4=u\sin\alpha \,.\, t-\frac{1}{2}gt^2 \qquad \ldots(2)$$

From (1),

$$t=\frac{12}{u\cos\alpha}.$$

Hence, from (2)

$$4=u\sin\alpha\,.\,\frac{12}{u\cos\alpha}-\frac{1}{2}\,g\,.\,\frac{144}{u^2\cos^2\alpha}$$

$$=12\tan\alpha-\frac{72\,g}{u^2\cos^2\alpha}$$

$$\Rightarrow\qquad 3\tan\alpha-\frac{18\,g}{u^2\cos^2\alpha}=1. \qquad (3)$$

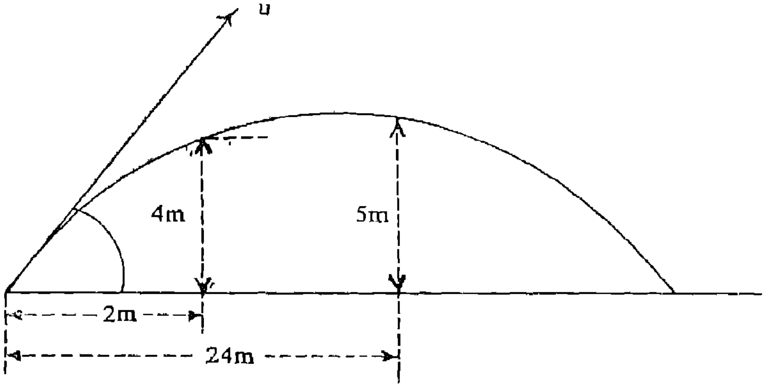


Fig 71

Similarly, considering the second position of the particle, we get

$$5=24\tan\alpha-\frac{288\,g}{u^2\cos^2\alpha}. \qquad (4)$$

From (3) and (4), we get

$$\frac{5-24\tan\alpha}{1-3\tan\alpha}=\frac{288}{18}=16$$

$$\Rightarrow\qquad 16-48\tan\alpha=5-24\tan\alpha$$

$$\Rightarrow\qquad 24\tan\alpha=11\quad\Rightarrow\quad\tan\alpha=\frac{11}{24}.$$

Hence, the direction of projection is $\tan^{-1}\frac{11}{24}$.

Now, from (3), we get

$$u^2=\frac{18\,g}{\cos^2\alpha\,(3\tan\alpha-1)}=\frac{18\times 9.8}{\dfrac{3\tan\alpha-1}{1+\tan^2\alpha}}$$

$$=\frac{18\times 9.8\times\left(1+\frac{121}{576}\right)}{}$$

$$=\frac{18\times 9.8\times 697\times 8}{576\times 3}$$

$$=\frac{34153}{60}.$$

$$\therefore\quad u=\left(\frac{34153}{60}\right)^{\frac{1}{2}}.$$

Hence, the velocity of projection is

$$\left(\frac{34153}{60}\right)^{\frac{1}{2}}\text{ m/s.}$$

Ex 5 A shot fired at a mark in the horizontal plane through the point of projection goes a m beyond it when the angle of elevation is α. When the angle of elevation is β, it falls b m short of the mark. Show that the proper elevation to hit the mark is $\frac{1}{2}\sin^{-1}\left(\frac{a\sin 2\beta+b\sin 2\alpha}{a+b}\right)$

Let θ be the proper elevation and x be the distance of the mark C from the point of projection O. Let u be the velocity of projection.

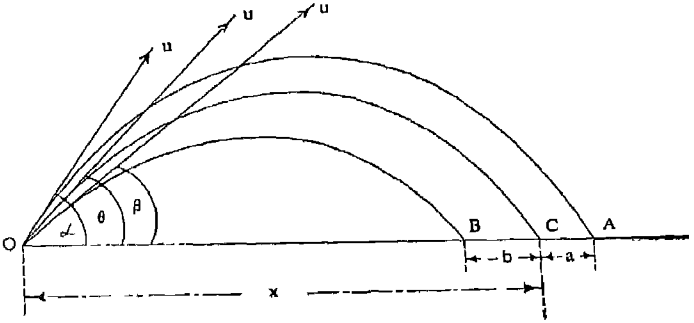


Fig 72

Then

$$x=\text{horizontal range}=\frac{u^2}{g}\sin 2\theta$$

$$\Rightarrow\qquad gx=u^2\sin 2\theta. \qquad \ldots(1)$$

In the second case, the angle of elevation is α and the horizontal range is $x+a$ corresponding to the point A.

Then $\qquad g(x+a)=u^2\sin 2\alpha.$

In the third case, the angle of elevation is β and the horizontal range is

$x-a$ corresponding to the point B.

$$\therefore \qquad g\,(x-b)=u^2 \sin 2\beta. \qquad (3)$$

Multiplying (2) by b and (3) by a and adding, we get

$$(a+b)\,gx=u^2\,(b \sin 2\alpha+a \sin 2\beta)$$

$$\Rightarrow \quad (a+b)\,u^2 \sin 2\theta=u^2\,(b \sin 2\alpha+a \sin 2\beta)$$

[From (1)]

$$\Rightarrow \qquad \sin 2\theta=\frac{a \sin 2\beta+b \sin 2\alpha}{a+b}$$

$$\Rightarrow \qquad \theta=\frac{1}{2}\sin^{-1}\left[\frac{a \sin 2\beta+b \sin 2\alpha}{a+b}\right].$$

Ex. 6. Two bodies projected simultaneously from a point with velocities v and v' at elevations α and α'. Show that the time between their passage through the point common to their paths is

$$\frac{2}{g}\cdot\frac{v\,v' \sin(\alpha-\alpha')}{v \cos \alpha+v' \cos \alpha'}.$$

Let t and t' be the times taken by the bodies projected with velocities v and v' respectively to reach the same point P whose co-ordinates are (x, y) with reference to rectangular axes through the point of projection.

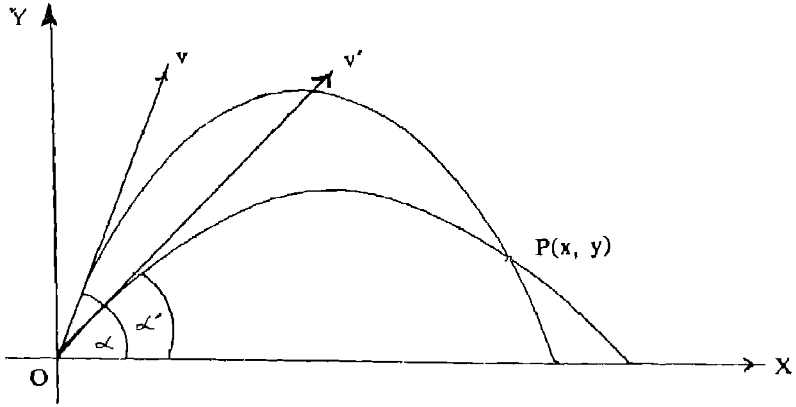


Fig. 73

Then considering the horizontal and vertical motions of the two bodies separately at the point (x, y), we get

$$x=v \cos \alpha \,.\, t=v' \cos \alpha' \,.\, t' \qquad \ldots(1)$$

and $$y=v\sin\alpha\, t-\frac{1}{2}gt^2=v'\sin\alpha'.t'-\frac{1}{2}gt'^2. \qquad (2)$$

From (1), we have

$$\frac{t}{v'\cos\alpha'}=\frac{t'}{v\cos\alpha}=k \text{ (say)}$$

$$\therefore \qquad t=kv'\cos\alpha',\ t'=kv\cos\alpha \qquad (3)$$

Now, from (2),

$$v\sin\alpha\, t-v'\sin\alpha'.t'=\frac{1}{2}g(t^2-t'^2)$$

$$\Rightarrow \quad k\,vv'\sin\alpha\cos\alpha'-kvv'\sin\alpha'\cos\alpha=\frac{1}{2}g(t-t')$$

$$\times k\,(v\cos\alpha+v'\cos\alpha')$$

[from (3)]

$$\Rightarrow \quad vv'\sin(\alpha-\alpha')=\frac{1}{2}g(t-t')(v\cos\alpha+v'\cos\alpha')$$

$$\Rightarrow \quad t-t'=\frac{2}{g}\cdot\frac{vv'\sin(\alpha-\alpha')}{v\cos\alpha+v'\cos\alpha'}.$$

Ex. 7 If R and R' be the ranges of two projectiles which being thrown from the same place attain the same vertical height and pass through a common point, then show that

$$RR'=\frac{4\,H\,h^2}{k},$$

where H is the greatest height attained and (h, k) is the common point of the two paths with reference to rectangular axes through the point of projection.

Let u be the velocity of projection and α be the angle of projection of the first projectile. Let P (x, y) be the position of the projectile at any time t. Then considering the vertical and horizontal motion of the projectile, we get

$$y=u\sin\alpha\, t-\frac{1}{2}gt^2 \qquad (1)$$

and $$x=u\cos\alpha.t. \qquad (2)$$

Eliminating t from (1) and (2), we get

$$y=u\sin\alpha.\frac{x}{u\cos\alpha}-\frac{1}{2}g.\frac{x^2}{u^2\cos^2\alpha}=x\tan\alpha-\frac{1}{2}.\frac{gx^2}{u^2\cos^2\alpha}.$$

Now, the horizontal range R is given by

$$R=\frac{u^2}{g}\sin 2\alpha, \qquad \ldots (3)$$

From (2) and (3), eliminating u^2, we get

$$y=x\tan\alpha-\frac{1}{2}\cdot\frac{gx^2\sin^2\alpha}{g\cos^2\alpha}$$

$$=x\tan\alpha-\frac{x^2\tan\alpha}{R}$$

$$=x\tan\alpha.\left(1-\frac{x}{R}\right) \qquad \ldots (4)$$

which is the equation of the first trajectory.

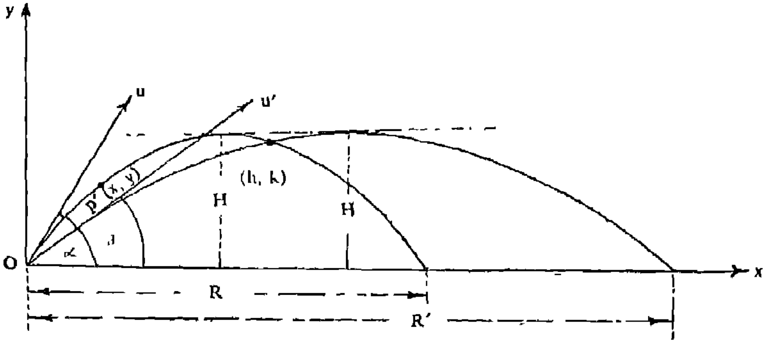


Fig. 74

Similarly, if β be the angle of projection of the second projectile, the equation of its trajectory is given by,

$$y=x\tan\beta.\left(1-\frac{x}{R'}\right). \qquad (5)$$

Now, the first projectile attains the maximum height H when $x=\frac{R}{2}$. Similarly, the second projectile attains the same maximum height H when $x=\frac{R'}{2}$.

Putting $x=\frac{R}{2}$ and $y=H$ in (4), we get

$$H=\frac{R}{2}\tan\alpha.\left(1-\frac{1}{2}\right)=\frac{R}{4}\tan\alpha. \qquad \ldots(6)$$

Similarly, putting $x=\frac{R'}{2}$ and $y=H$ in (5), we get

$$H=\frac{R'}{4}\tan\beta \qquad \ldots(7)$$

Since (h, k) lies on both the curves (4) and (5), we have

$$k=h\tan\alpha.\left(1-\frac{h}{R}\right)=\frac{4h\,H}{R}\left(1-\frac{h}{R}\right) \qquad \ldots(8)$$

[from (6)]

and $k=h\tan\beta.\left(1-\frac{h}{R'}\right)=\frac{4hH}{R'}\left(1-\frac{h}{R'}\right).$

[from (7)]

Hence, $\frac{4h\,H}{R}\left(1-\frac{h}{R}\right)=\frac{4hH}{R'}\left(1-\frac{h}{R'}\right)$

$$\Rightarrow \quad \frac{1}{R}-\frac{h}{R^2}=\frac{1}{R'}-\frac{h}{R'^2}$$

$$\Rightarrow \quad \frac{1}{R}-\frac{1}{R'}=h\left(\frac{1}{R^2}-\frac{1}{R'^2}\right)$$

$$\Rightarrow \quad \frac{R'-R}{RR'}=h.\frac{R'^2-R^2}{R^2\,R'^2} \quad \Rightarrow \quad 1=\frac{h\,(R'+R)}{RR'}$$

$$\Rightarrow \quad 1=\frac{h}{R}+\frac{h}{R'}$$

$$\Rightarrow \quad 1-\frac{h}{R}=\frac{h}{R'}. \qquad \ldots(9)$$

From (8) and (9), we get

$$k=\frac{4\,h^2\,H}{RR'}$$

$$\Rightarrow \quad RR'=\frac{4\,h^2\,H}{k}.$$

Ex 8. The angular elevation of an enemy's position on a hill s m above the gun position is β. Show that in order to shell it, the velocity of the projectile must not be less than

$$\sqrt{g\,s\,(1+\operatorname{cosec}\beta)}$$

[C.U. '46]

Let O be the position of the gun, A, the enemy's position on the hill, $AB=s$ m and OB be horizontal. Here $\angle AOB=\beta$. Let u be the velocity of projection and α be the angle of projection. Let the projectile reach A in time t. Now, $OB=s\cot\beta$.

Then considering the horizontal and vertical motion of the projectile, we get

$$u \cos \alpha .\ t = s \cos \beta \tag{1}$$

and $$u \sin \alpha .\ t = -\frac{1}{2}\ g t^2 = s$$

$$\Rightarrow \qquad u \sin \alpha \ \ t = s + \frac{1}{2}\ g t^2 \tag{2}$$

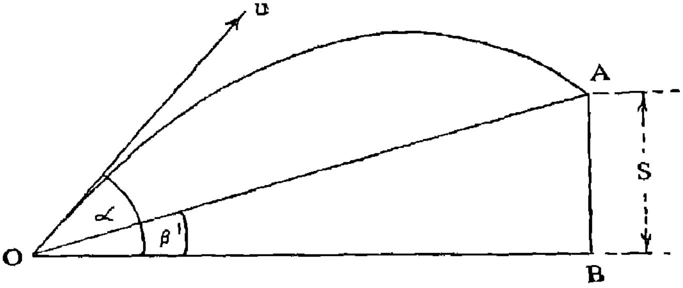


Fig. 75

Eliminating α by squaring (1) and (2) and adding, we get

$$u^2\, t^2 = s^2\ \cot^2 \beta + \left(s + \frac{1}{2}\ g t^2 \right)^2$$

$$= s^2\ \cot^2 \beta + s^2 + \frac{1}{4}\ g^2\, t^4 + g\ s t^2$$

$$\Rightarrow \qquad \frac{1}{4}\ g^2\ t^4 - (u^2 - g s)\ t^2 + s^2\ \operatorname{cosec}^2 \beta = 0.$$

This is a quadratic equation in t^2 Since t^2 should be always a real positive number, we must have

$$(u^2 - gs)^2 \geqslant g^2\ s^2\ \operatorname{cosec}^2\ \beta \qquad \ldots(3)$$

and $$u^2 - gs > 0. \qquad (4)$$

The equations (3) and (4) imply that

$$u^2 - gs \geqslant gs\ \operatorname{cosec}\ \beta$$

$$\Rightarrow \qquad u^2 \geqslant gs\ (1 + \operatorname{cosec}\ \beta)$$

$$\Rightarrow \qquad u \geqslant gs \sqrt{1 + \operatorname{cosec}\ \beta}$$

$[\because \ \ u > 0]$

Hence the result

Ex. 9. If v_1 and v_2 be the velocities of a projectile at the ends of a focal chord of its path, and v be the constant horizontal velocity, show that $\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{v^2}$

Let s be the focus of the parabolic path of the projectile and P_1SP_2 be a focal chord of the parabola. V_1 is the velocity of the projectile at P_1 in the direction of the tangent to the curve at P_1 and v_2 is the velocity of the projectile at P_2 in the direction of the tangent to the curve. Let these two tangents intersect at T. From the properties of the parabola, we know that these two tangents will intersect at right angles on the directrix of the parabola. Let P_1T make an angle φ with the vertical TM through T. Hence, P_2T will make an angle $90°-\varphi$ with this vertical. If A be the highest point on the parabola, then the velocity v at A is horizontal.

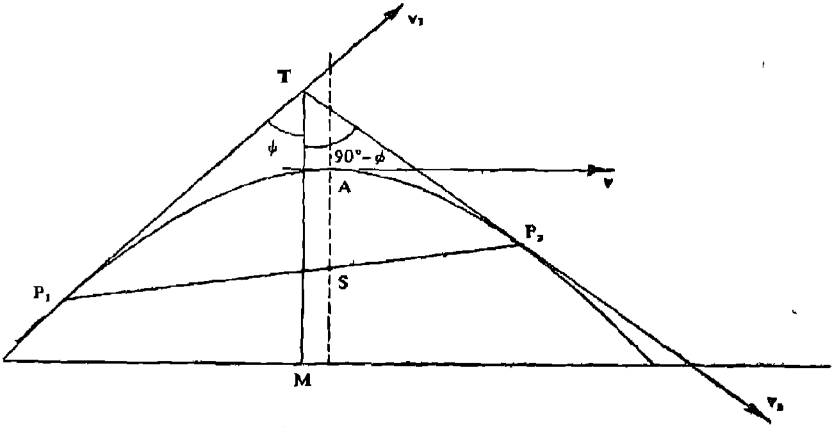


Fig. 76

Now, since in the horizontal direction, the velocity remains constant, we have

$v_1 \sin\varphi$=horizontal component of the velocity at $P_1=v$

$$\Rightarrow \qquad \sin\varphi=\frac{v}{v_1}=v \qquad ..(1)$$

and $v_2 \sin(90°-\varphi)$=horizontal component of the velocity at $P_2=v$

$$\Rightarrow \qquad v_2\cos\varphi=v$$

$$\Rightarrow \qquad \cos\varphi = \frac{v}{v_2} \qquad \ldots(2)$$

Squaring (1) and (2) and adding, we get

$$1 = v^2\left(\frac{1}{v_1^2} + \frac{1}{v_2^2}\right)$$

$$\Rightarrow \qquad \frac{1}{v_1^2} + \frac{1}{v_2^2} = \frac{1}{v^2}$$

Ex. 10. A regular hexagon stands with one side on the ground and a particle is projected so as just to graze its four upper vertices Show that the velocity of the particle on reaching the ground to its least velocity as $\sqrt{31}$ to $\sqrt{3}$.

Let *ABCDEF* be a regular hexagon of side $2a$ and let the side *AB* lie on the ground. A particle is projected from a point *A* on the ground with a minimum velocity so as to graze the vertices *F*, *E*, *D* and *C* of the hexagon.

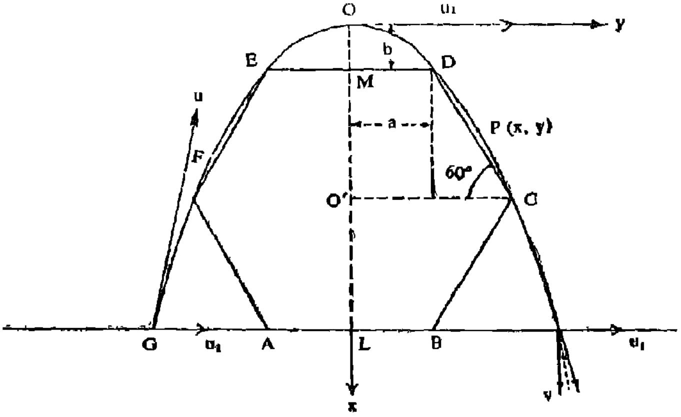


Fig 77

Let u_1 be the horizontal component of the velocity of projection, u, from G. Since the horizontal component u_1 of u remains constant throughout the motion of the particle, u will be minimum only when the particle is at the highest point of its path where the vertical component of the velocity becomes zero and hence the minimum value of u is u_1. Let O be the highest point of the parabolic path. If L and M be the mid-points of *AB* and *ED* respectively, then from symmetry, *LM* will pass through O, *AB* vertex of the parabola. We choose *OML* as the x-axis. Let $OM=b$. At O, the velocity of the particle is u_1 along *OY*. Its vertical component at

O is zero. If P (x, y) be any point on the curve where the particle reaches from O in time t, then considering the horizontal and vertical motion of the particle, we have

$$y=u_1 t \text{ and } x=\frac{1}{2} gt^2.$$

Eliminating t, we get

$$y^2=\frac{2u_1^2}{g} x, \qquad \ldots(1)$$

which is the equation of the path of the particle.

Now, $\quad OM=b$ and $MD=a$

Hence, the co-ordinates of D are (b, a).

If O' be the centre of the hexagon, then $OO'=OM+MO'$

$$=b+2a \sin 60^\circ$$

$$=b+\sqrt{3}\, a.$$

$$O'C=MD+2a \cos 60^\circ=a+a=2a.$$

Hence, the co-ordinates of C are $(b+a\sqrt{3},\ 2a)$.

Now, both D and C lie on the parabola (1). Hence, putting $x=b$ and $y=a$, and $x=b+a\sqrt{3}$, $y=2a$ in (1) successively, we get

$$a^2=\frac{2u_1^2\, b}{g} \qquad \ldots(2)$$

and

$$4\, a^2=\frac{2u_1^2\, b}{g}(1+a\sqrt{3}\,).$$

Eliminating u_1^2, we get

$$\frac{1}{4}=\frac{b}{b+a\sqrt{3}}\,.$$

$$\Rightarrow \quad b+a\sqrt{3}=4\, b \quad \Rightarrow \quad 3b=a\sqrt{3} \quad \Rightarrow \quad b=\frac{a}{\sqrt{3}} \qquad \ldots(3)$$

$\therefore$ From (2), we have

$$u_1^2=\frac{ga^2}{2b}=\frac{ga^2\sqrt{3}}{2a}=\frac{ag}{2}\sqrt{3}\,. \qquad (4)$$

Now, $OL=OM+ML=b+3\times 2a \sin 60^\circ$

$$=\frac{a}{\sqrt{3}}+2a\sqrt{3} \quad \text{[from (3)]}$$

$$=\frac{7a}{\sqrt{3}}.$$

Now, considering the vertical motion from the highest point to the ground, we get

$$v^2 = 2g.\ OL = 2g.\frac{7a}{\sqrt{3}} = \frac{14\,ag}{\sqrt{3}}, \qquad \text{(5)}$$

where v is the vertical component of the velocity of the particle on reaching the ground The horizontal component of the velocity of the particle on reaching the ground is u_1. If V be the resultant velocity on reaching the ground, we have

$$\begin{aligned} V^2 &= v^2 + u_1^2 \\ &= \frac{14\,ag}{\sqrt{3}} + \frac{\sqrt{3}\,ag}{2} \qquad \text{[from (4) and (5)]} \\ &= \frac{31\,ag}{2\sqrt{3}} = \frac{31\sqrt{3}\,ag}{6}. \qquad \text{(6)} \end{aligned}$$

From (4) and (6), we get

$$\begin{aligned} V^2 : u_1^2 &= \frac{31}{6} : \frac{1}{2} \\ &= 31 : 3 \end{aligned}$$

$$\therefore \qquad V : u_1 = \sqrt{31} \cdot \sqrt{3}\ \cdot$$

(∵ both V and u_1 are positive)

Ex. 11. A shell bursts on contact with the ground and pieces from it fly in all directions with all velocities upto 25 m/s. Find the time during which a man 30 m away will be in danger.

Let u m/s be the velocity of projection of a shot, making an angle θ with the horizon. Let it hit the man M at a distance 30m from O, the point of projection after t s. Then considering the horizontal and vertical motion of the shot, we get

$$u\cos\theta\,.t = 30 \qquad \text{(1)}$$

and

$$u\sin\theta\,.\,t - \frac{1}{2}\,gt^2 = 0 \qquad \text{..(2)}$$

Eliminating θ from (1) and (2), we get

$$\frac{30}{u^2t^2} + \frac{\left(\frac{1}{2}\,gt^2\right)^2}{u^2\,t^2} = 1$$

$$\frac{1}{4}\,g^2\,t^4 - u^2\,t^2 + 30^2 = 0$$

$$\therefore \qquad t^2=\frac{u^2\pm\sqrt{u^4-30^2\,g^2}}{\frac{1}{2}\,g^2}$$

If $u^4 \geqslant 30^2\,g^2 \Rightarrow u \geqslant 30\,g$, both the values of t^2 are real Hence, there are two pieces of the shot, each moving with any velocity u m/s, to hit the man at two different times t_1 and t_2 $(t_1<t_2)$ given by

$$t_1^2=\frac{u^2-\sqrt{u^4-30^2\,g^2}}{\frac{1}{2}\,g^2} \quad \cdots (3)$$

and

$$t_2^2=\frac{u^2+\sqrt{u^4-30^2\,g^2}}{\frac{1}{2}\,g^2}$$

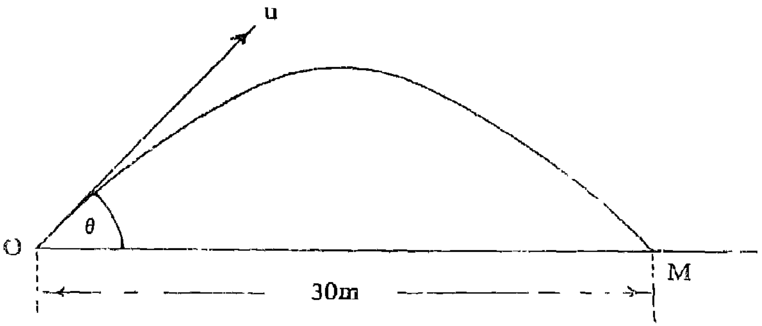


Fig. 78

Since $t_2>t_1$, we see that for different velocities of projection, the latest time t_2 at which a piece of the shot can hit the man is when u is greatest i.e. when $u=25$ m/s. Hence, the greatest value of t_2 is given by

$$t_2^2=\frac{25^2+\sqrt{25^4-900\times 9.8^2}}{\frac{1}{2}\times 9.8^2}$$

$$=\frac{625+\sqrt{304189}}{48.02}\simeq\frac{1176.53}{48.02}=24.50$$

$$\Rightarrow \qquad t_2=4.95.$$

Thus the latest time at which a piece of the shot can hit the man is 4.95s nearly. After this time, no piece of the shot can hit the man.

Now, (3) can be written as

$$t_1^2=\frac{30^2\,g^2}{\frac{1}{2}\,g^2\,(u^2+\sqrt{u^4-30^2\,g^2}\,)}$$

$$=\frac{2\times30^2}{u^2+\sqrt{u^4-30^2\,g^2}}\,.$$

Thus we see that t_1 is least when u is greatest i.e. when $u=25$ m/s. Thus, of all the pieces that hit the man, the earliest time when a piece can hit the man is given by the minimum value of t_1 which is given by

$$t_1^2=\frac{25^2-\sqrt{25^4-30^2\times9.8^2}}{\frac{1}{2}\times9.8^2}$$

$$=\frac{625-\sqrt{304189}}{48.02}$$

(as in the provious case)

$$\simeq\frac{73\,47}{48.02}\simeq1.53$$

$$\therefore\qquad t_1\simeq1\,24.$$

Thus the earliest time at which a piece of the shot can hit the man is 1.24s nearly. Before this time, no piece can hit the man.

Hence, the man remains in danger for (t_2-t_1) s or $(4.95-1.24)$ s i.e. for 3.71s nearly.

Ex. 12 Particles are projected from a given point O with velocities of the same magnitude u but in different directions in the same vertical plane. Prove that at time t after projection, these will lie on a circle of radius ut.

Let O be the point of projection and α be the angle of projection of a particle. We choose the horizontal and vertical through O as the x and y-axes. Let $P\,(x,\,y)$ be the position of the particle at any time t (See Fig. 65). Then considering the horizontal and vertical motion of the particle, we get

$$x=u\cos\alpha\,.\,t$$

and $$y=u\sin\alpha\,.\,t-\frac{1}{2}\,gt^2.$$

Eliminating α, we get

$$\left(\frac{x}{ut}\right)^2+\frac{\left(y+\frac{1}{2}\,gt^2\right)^2}{u^2\,t^2}=1$$

$$\Rightarrow \qquad x^2+\left(y+\frac{1}{2}gt^2\right)^2=u^2t^2$$

which represents a circle with centre $\left(0, -\frac{1}{2}gt^2\right)$ and radius ut

Hence the result.

Ex. 13. Show that the greatest height which a shot fired with initial velocity u can reach on a vertical wall at a distance d from the point of projection is

$$\frac{u^2}{2g}-\frac{gd^2}{2u^2}.$$

Let α be the angle of projection of the shot and t, the time to reach a height h of the wall which is at a horizontal distance of d from the point of projection

Considering the horizontal and vertical motion of the shot, we get

$$d=u\cos\alpha\,.\,t \qquad \ldots(1)$$

and

$$h=u\sin\alpha\,.\,t-\frac{1}{2}gt^2 \qquad \ldots(2)$$

Substituting $t=\dfrac{d}{u\cos\alpha}$, obtained from (1) in (2), we get

$$h=d\tan\alpha-\frac{1}{2}g.\frac{d^2}{u^2}\sec^2\alpha. \qquad \ldots(3)$$

We shall now find α for which h is maximum Differentiating both sides of (3) with respect to α, we get

$$\frac{dh}{d\alpha}=d\sec^2\alpha-\frac{1}{2}g.\frac{d^2}{u^2}\;2\sec^2\alpha\tan\alpha$$

$$=d\sec^2\alpha\left(1-\frac{gd}{u^2}\tan\alpha\right).$$

For maximum or minimum,

$$\frac{dh}{d\alpha}=0$$

$$\Rightarrow \qquad \tan\alpha=\frac{u^2}{gd}.$$

Also,

$$\frac{d^2h}{d\alpha^2}=2d\sec^2\alpha\tan\alpha\left(1-\frac{gd}{u^2}\tan\alpha\right)$$

$$+d\sec^2\alpha\left(-\frac{gd}{u^2}\sec^2\alpha\right)$$

$$=d\sec^2\alpha\left(2\tan\alpha-\frac{3gd}{u^2}\tan^2\alpha-\frac{gd}{u^2}\right)$$

$\therefore$ when $\tan\alpha=\dfrac{u^2}{gd}$, we have

$$\frac{d^2h}{d\alpha^2}=d\left(1+\frac{u^4}{g^2d^2}\right)\left[\frac{2u^2}{gd}-\frac{3u^2}{gd}-\frac{gd}{u^2}\right]$$

$$=-d\left(1+\frac{u^4}{g^2d^2}\right)\left(\frac{u^2}{gd}+\frac{gd}{u^2}\right)$$

which is obviously negative

Hence, h is maximum when $\tan\alpha=\dfrac{u^2}{gd}$.

Putting $\tan\alpha=\dfrac{u^2}{gd}$ in (3), we get the maximum value of h as

$$\frac{u^2}{g}-\frac{1}{2}g\frac{d^2}{u^2}\left(1+\frac{u^4}{g^2d^2}\right)$$

$$=\frac{u^2}{g}-\frac{1}{2}\frac{gd^2}{u^2}-\frac{1}{2}\frac{u^2}{g}=\frac{u^2}{2g}-\frac{gd^2}{2u^2}.$$

Ex. 14. Find when the particle which is projected with velocity u at an inclination α to the horizontal will be moving in a direction at right angles to the initial direction and find the velocity at that moment.

Let u be the velocity of projection and α be the angle of projection of the particle. At any time t, let the position of the particle be at P. Let v be the velocity of the particle at P making an angle θ with the horizontal.

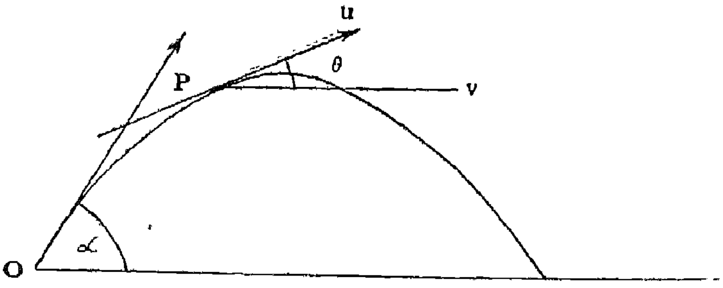


Fig 79

Then considering the horizontal and vertical motion of the particle, we get

$$v \cos\theta = u \cos\alpha \qquad \ldots(1)$$

and $$v \sin\theta = u \sin\alpha - gt \qquad \ldots(2)$$

Squaring (1) and (2), and adding, we get

$$v^2 = u^2 \cos^2\alpha + u^2 \sin^2\alpha - 2u\,gt \sin\alpha + g^2 t^2$$

$$= u^2 - 2ugt \sin\alpha + g^2 t^2$$

$$\Rightarrow \qquad v = \pm\sqrt{u^2 - 2ugt \sin\alpha + g^2 t^2}. \qquad \ldots(3)$$

Also, dividing (2) by (1), we get

$$\tan\theta = \frac{u \sin\alpha - gt}{u \cos\alpha}. \qquad \ldots(4)$$

Now, the direction θ of the velocity v at P will be perpendicular to the direction α of projection when

$$\theta = \alpha \pm 90^\circ.$$

Putting $\theta = \alpha \pm 90^\circ$ in (4), we get

$$-\cot\alpha = \frac{u \sin\alpha - gt}{u \cos\alpha}$$

$$\Rightarrow \qquad \frac{-u \cos^2\alpha}{\sin\alpha} = u \sin\alpha - gt$$

$$\Rightarrow \qquad u \cos^2\alpha + u \sin^2\alpha = gt \sin\alpha$$

$$\Rightarrow \qquad t = \frac{u}{g \sin\alpha}.$$

Hence, the particle will be moving at right angles to the initial direction after time $t = \frac{u}{g \sin\alpha}$. Then the magnitude of the velocity of of the particle will be

$$\sqrt{u^2 - 2\,ug \sin\alpha . \frac{u}{g \sin\alpha} + g^2 . \frac{u^2}{g \sin^2\alpha}} \qquad \text{[from (3)]}$$

$$= \sqrt{u^2 - 2u^2 + u^2 \operatorname{cosec}^2\alpha}$$

$$= u\sqrt{\operatorname{cosec}^2\alpha - 1} \qquad = u \cot\alpha.$$

Ex. 15. Show that, if R be the maximum horizontal range for a given velocity of projection, a particle can be projected to pass through a point whose horizontal and vertical distances from the point of projection are $\frac{1}{2}R$ and $\frac{1}{4}R$ respectively, provided that the tangent of the angle of projection is 1 or 3, and that in the second case, the range in the horizontal plane is $\frac{3}{5}R$.

Let u be the velocity of projection and α be the angle of projection. Then we know that the maximum horizontal range R is given by

$$R=\frac{u^2}{g}. \tag{1}$$

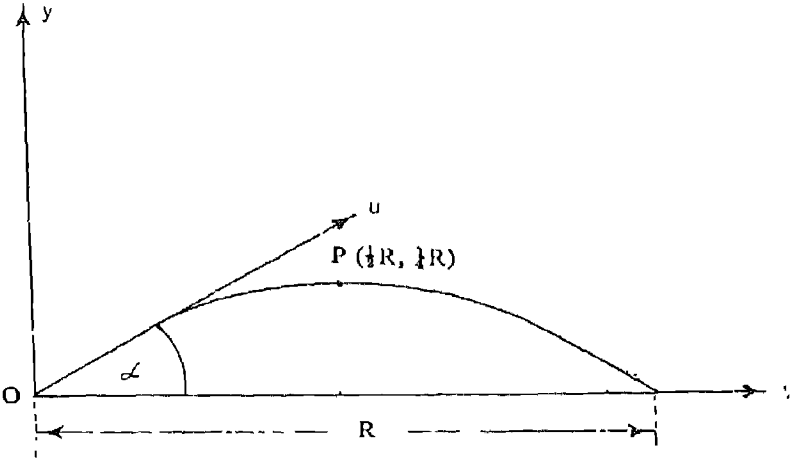


Fig. 80

Referred to rectangular axes through O, the point of projection, the equation of the trajectory is given by

$$y=x\tan\alpha-\frac{g}{2u^2}\cdot x^2\sec^2\alpha. \qquad \ldots (2)$$

Since $\left(\frac{1}{2}R, \frac{1}{4}R\right)$ is a point on this curve, we have

$$\frac{1}{4}R=\frac{1}{2}R\tan\alpha-\frac{g}{2u^2}\times\frac{1}{4}R^2(1+\tan^2\alpha)$$

$$\Rightarrow \qquad =\frac{1}{4}=\frac{1}{2}\tan\alpha-\frac{1+\tan^2\alpha}{8}$$

$\Rightarrow \quad 2=4\tan\alpha-(1+\tan^2\alpha)$

$\Rightarrow \quad \tan^2\alpha-4\tan^2\alpha+3=0$

$\Rightarrow \quad (\tan\alpha-1)(\tan\alpha-3)=0$

$\Rightarrow \quad \tan\alpha=1$ or 3.

Now, the horizontal range R' is given by

$$R'=\frac{u^2}{g}\sin 2\alpha=\frac{2u^2}{g}\sin\alpha\cos\alpha$$

$$=2R\sin\alpha\cos\alpha.$$

[from (1)]

Now, when $\tan\alpha=3$, then $\sin\alpha=\frac{3}{\sqrt{10}}$ and $\cos\alpha=\frac{1}{\sqrt{10}}$.

Hence, in this case

$$R'=2R.\frac{1}{\sqrt{10}}\times\frac{1}{\sqrt{10}}=\frac{3}{5}R.$$

Hence the result.

EXERCISE ON CHAPTER 8

1. A shell is projected with a velocity of 100 m/s at an angle of 45° to the horizontal. Calculate the greatest height reached and the horizontal range. Also calculate the velocity of the shell when $t=1$ s. [Ans. 255.1 m, 1020.4 m, 93.31 m/s making an angle of $\tan^{-1}\left(1-\frac{49}{250\sqrt{2}}\right)$ with the horizontal]

2. A man and his dog are standing side by side on a horizontal plane when the man throws a ball with velocity 5 m/s at an angle of 30° to the horizontal. What is the average speed of the dog who sets off immediately and catches the ball just before it hits the ground ? [Ans. $\frac{5\sqrt{3}}{2}$ m/s]

3. A boy stands on a cliff of height 100 m above sea-level and throws a stone out to a sea with an initial velocity of 15 m/s in a

horizontal direction. How long does the stone take to reach the sea ? [Ans. 4.518 s]

4. Find the angle of projection which gives a body a horizontal range of 8 km when projected with a speed of 400 m/s. $\left[\text{Ans. } \frac{1}{2} \sin^{-1}(49)\right]$

5. A gun which can fire a shell with a speed of 400 m/s in any direction is placed in a bunker on a horizontal plane Find the maximnm area which the gunner can command. [Ans.$(16326.53)^2 \pi$ m^2]

6. A coastguard standing on a cliff 100 m above sea-level is trying to fire a life-line to a boat on the sea below. The firing mechanism fires the line bolt with a velocity of 75 m/s horizontally. Calculate how far out the line will reach if 10 m short is allowed for the drag of the line. [Ans. 329 m nearly]

7. A jet of water is ejected with a speed of 20 m/s from a fire hose inclined at 45° to the horizontal. Calculate how far the water will reach on the horizontal ground, assuming no air resistance is offered to the water.

   At what angle must the hose be inclined in order to direct the same jet of water horizontally through a window at a height of 4.9 m in a vertical wall ? [Ans. 40.8 m, $\sin^{-1}(.49)$]

8. A body is projected with a velocity u at an elevation α. Calculate the velocity of the body when it has reached a height y above the point of projection [Ans. $\sqrt{u^2-2gy}$]

9. A cricketer is standing 50 m from the batsman when he catches him out. He estimates that the ball comes to him at an angle of 30° to the horizontal. Calculate the velocity with which the ball leaves the bat. (Assume that the ball is caught just above the grouud). [Ans 23.77 m/s]

10. A diver on a fixed high board runs forward with a velocity of 2 m/s. How far forward will he strike the water 10 m below and how long will he be in the air ? If he now returns to the high spring board 10 m above the water-level, how much longer will he be in the air in order to execute a somersault and twist by leaping up with a velocity of 2 m/s at 80° to the horizontal ? ($\sin 80° = 0.985$) $\left[\text{Ans. } 2\frac{6}{7} \text{ m; } 1\frac{3}{7} \text{ s; } 0.22\text{s nearly}\right]$

11. Prove that the trajectory of a projectile which gives the maximum horizontal range R also has a greatest height of $\frac{1}{4}R$.

12. A ship is sailing westward with a velocity of 8 m/s. A sailor drops a bolt from the top of a must. If the mast of the ship is 19.5 m high, where will the bolt hit the deck ? Consider two cases when (i) the ship stops moving immediately after the bolt is dropped, and (ii) the ship continues to move even after the bolt is dropped. [Ans. (i) 16 m away from the foot of the mast (ii) Exactly at the foot of the mast]

13. From the top of a building, 19.6 m high, a ball is projected horizontally. After how long does it strike the ground ? If the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal, what is the initial velocity of the ball ? [Ans. 2 s, 9.8 m/s]

14. A bomb is dropped from an aeroplane when it is directly above a target at a height of 1500 m. The aeroplane is going horizontally with a speed of 500 km/h. By how much distance will the bomb miss the target ?

$$\left[\text{Ans. } \frac{62500\sqrt{6}}{63}\text{ m}\right]$$

15. A foot-ball is punted at an angle of 60° above the horizontal. Find

(i) the velocity with which it leaves the punter's toe if it travels 20 m, measured horizontally on the field.
(ii) to what maximum height the ball will rise.
(iii) the time of flight.

[Ans. (i) 15.04 m/s (ii) 8.66 m (iii) 2.658 s]

16. A ball is thrown horizontally with a velocity of 15 m/s from a tower 30m high. Find the time of flight, the horizontal range and the speed of the ball just before it strikes the ground. [Ans. 2.47 s, 37.05 m, 24.25 m/s]

17. A plane is climbing at an angle of 30° above the horizontal at a speed of 400 m/s. When it is at an altitude of 1595 m, it releases a bomb. Find

(i) in how many seconds the bomb reaches the ground.

(ii) how for beyond the point of release does the bomb strike.

(Take $g=10\text{ m/s}^2$)

[Ans. (i) 59s (ii) 10045.89 m]

18. An aeroplane is flying in a horizontal direction with a velocity of 600 km/h and at a height of 1960 m. When it is vertically above a point A on the ground, a body is dropped from it. The body strikes the ground at a point B Neglecting air resistance, calculate the distance AB. [I.I.T. '63]
[Ans. 3.33 km approx.]

19. A shot fired from a gun passes horizontally just over a wall. If the wall be 19.6 m high and 58.8 m distant from the gun, find the direction and the velocity of projection of the shot
$$\left[\text{Ans. } \tan^{-1}\frac{2}{3},\ 35.33 \text{ m/s approx.}\right]$$

20. From the top of a tower, 63.7 m high, a stone is projected, the horizontal and the vertical components of the velocity being 78.4 m/s and 58.8 m/s respectively Find (i) the time of flight and (ii) the distance of the point where it strikes the ground, from the foot of the tower. [Ans. (i) 13 s (ii) 1019.2 m]

21. A batsman hits a ball just where it is pitched. The ball leaves the ground at an angle of 37° with the horizontal, and with an initial speed of 14.7 m/s. A fielder standing at a distance of 29.4 m from the batsman in the direction of the shot starts running at a constant speed to catch the ball at the instant the ball is hit. How fast must he run to just make the catch ?
($\sin 37^\circ = .60$) [Ans. 4.57 m/s approx.]

22. What is the least velocity with which a cricket ball can be thrown 80 m horizontally ? [Ans. $\sqrt{80g}$ m/s]

23. If the maximum horizontal range for a particle is R, show that the greatest height attained is .25 R.

24. A particle is projected with a velocity V_0 so that its range on a horizontal plane is twice the greatest height attained. Show that the range is .0816 V_0^2.

25. Find the maximum horizontal range when the velocity of projection is 28 m/s. [Ans. 80 m]

26. A ball projected with a velocity of 28 m/s has a horizontal range of 40 m. What is the angle of projection ? [Ans. 15° or 75°]

27. A bullet fired at an angle of θ to the horizontal has a horizontal range R metres and time of flight T seconds. Show that
$$\tan\theta = \frac{4.9\,T^2}{R}.$$

28. If R is the horizontal range of a projectile and H, the greatest height, prove that the initial velocity V_0 is given by

$$V_0=\left[2g\left(H+\frac{R^2}{16\,H}\right)^{\frac{1}{2}}\right].$$

29. The initial horizontal and vertical components of a projectile are V_{ox} and V_{oy} respectively. If R is the horizontal range and H, the greatest height attained, show that

(i) $\frac{4H}{R}=\frac{V_{oy}}{V_{ox}}$ (ii) $\frac{8H}{g}=\left[\frac{R}{V_{ox}}\right]^2$

(iii) the maximum horizontal range is

$$2H+\frac{R^2}{8H}.$$

30. If t_1 be the time in which a projectile reaches a point P in its path and t_2, the time from P till it reaches the horizontal plane through the point of projection, show that the height of P above the horizontal plane is $\frac{1}{2}g t_1\,t_2$.

31. A cannon ball has a range R on a horizontal plane. If H, H' are the greatest heights and t, t', the times of flight in the two possible paths, show that

$$R=\sqrt{4HH'}=\frac{1}{2}gt\,t'.$$

32. Two balls are projected from the same point in directions inclined at 60° and 30° to the horizontal. (i) If they attain the same height, what is the ratio of their velocities of projection ? (ii) If they have the same horizontal range, what is the ratio of their velocities of projection ? [Ans. (i) $1:\sqrt{3}$ (ii) $1:1$]

33. Show that a given gun will shoot three times as high when elevated at an angle of 60° as when fired at an angle of 30°, but will carry the same distance on a horizontal plane.

34. A body is projected vertically upwards from a point A with a velocity u. If θ be the direction in which another body must be projected simultaneously with velocity V_0 from a point B in the same horizontal line with A so as to strike the first body is given by

$$\sin\theta=\frac{u}{V_0}.$$

35. A particle is projected with velocity V_0 at an evevation θ. Prove that if $h < \frac{V_0^2 \sin^2 \theta}{2g}$, then $\frac{2}{g}\sqrt{V_0^2 \sin^2 \theta - 2gh}$ is the time required by the particle to pass through two points having the same height h.

36. A cricketer can throw a ball 49 m vertically upwards. How far, at the most, can he throw it on a level field ? [Ans. 98 m]

37. If V_0 be the velocity and θ be the angle of projection of a body, show that the body will be moving perpendicular to its original direction of motion after a time $\frac{V_0}{g}$ cosec θ.

38. Two seconds after its projection, a projectile is travelling in a direction inclined at 30° to the horizon and after one more second, it is travelling horizontally. Determine the velocity and the angle of projection, [Ans. 33 95 m/s, 60°]

39. A bomb was released from an aeroplane when it was at a height of 1000 m above a point A on the ground and was moving horizontally with a speed of 100 m/s. Find the distance from A of the point where the bomb strikes the ground.

[Ans. 1414.21 m]

40. A shell bursts on contact with the ground and the pieces from it fly in all directions with all speeds upto 30 m/s. Show that a man 30 m away is in danger for nearly 5 s.

41. A stone is thrown up with a velocity of 39 2 m/s at 30° to the horizontal. Find at what times, it will be at a height of 14.7 m

[Ans. 1 s and 3 s]

42 When a bullet is fired in a direction making an angle θ with the horizontal, the range is 1000 m. If the bullet is fired with the same angle from a car travelling with a speed of 36 km/h towards the target, show that the range will be increased by $\frac{1000}{7}\sqrt{\tan \theta}$.

43. A particle is projected so as to graze the tops of two walls, each of height 20 m at distances 30 m and 170 m respectively from the point of projection. Find the angle of projection.

[Ans. 38°6′]

44. A gun kept on a straight horizontal road is used to hit a car travelling away from the gun with a uniform speed of 72 km/h. The car is at a distance of 500 m from the gun, when the gun is fired at an angle of 45° with the horizontal Find (i) the distance of the car from the gun when the shell hits it and (ii) the speed of projection of the shell from the gun. (Neglect the height of the car) [I.I.T '74]

[Ans. (i) 746.9 m (ii) 85.56 m/s]

45. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β are the base angles, and θ, the angle of projection, show that $\tan\theta=\tan\alpha+\tan\beta$

46. A revolver can fire a bullet with a muzzle velocity of 63 m/s. Is it possible to hit the top of a tower, 400 m away, its height being 30 m ? [Ans No.]

47. A block of ice starts sliding down from the top of the inclined roof of a house along a line of maximum slope, the angle of inclination of the roof being 30° with the horizontal. The highest and the lowest points of the roof are at heights 8.1 m and 5 6 m respectively from the ground. At what horizontal distance from the starting point will the block of the ice hit the ground ? (Neglect friction) [I.I.T.' 72]

[Ans. 8.998 m]

48 A stone is projected from a point on the ground in such a direction α with the horizontal as to hit a bird on the top of a telegraph post of height h and then attains a maximum height $2h$ above the ground If, at the instant of projection, the bird were to fly away horizontally with a uniform speed, find the ratio between the horizontal velocity of the bird and the velocity of the stone, if the stone still hits the bird while descending

[Ans. $\cos\alpha:1$]

49. A particle is projected with a velocity of 29.4 m/s at an angle of 60° to the horizontal. Find the range on a plane inclined at an angle of 30° to the horizontal when projected up the plane.

[Ans. 58.8 m]

50. Two paper screens, A and B, are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is .1 m below the hole in A. If the bullet is travelling horizontally at

the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. (Neglect the resistance of the paper and air.) [I I T. '70]

[Ans. 700 m/s]

51. A pilot flying at 300 km/h along a horizontal line 300 m above the ground level wishes to bomb a tank travelling at 30 km/h in the same direction on the ground. How many seconds before he is directly over the tank must he release the bomb and what must be the distance measured in the horizontal direction, separating the plane and the tank? [Ans. 7.825 s, 586.8 m]

52. A boy throws a ball with a velocity of 16 m/s at an angle of 45° to the horizontal. The ball strikes a wall at a distance of 3 m from the body. When will the ball strike the wall when the ball ascends or descends? Find the height at which the ball will strike the wall. Determine the velocity of the ball at the moment of impact. [Ans. After $\frac{3\sqrt{2}}{10}$ s during ascend; 2.118 m above the boy; 7.64 m/s]

53. A particle is projected so as to graze the tops of two walls, each of height 7 m at distances 10 m and 50 m respectively from the point of projection. Find the angle of projection.

$$\left[\text{Ans.}\ \tan^{-1}\frac{21}{25}\right]$$

54. A shell breaks into two equal parts at the highest point of its path, 19.6 m above the ground and at a horizontal distance of 1000 m from the point of firing. Two seconds after the explosion, one of the pieces, say the first, hits the ground at a point vertically below the point where the shell broke. Calculate the velocity of (i) the shell before breaking, (ii) the first part immediately after the shell broke, (iii) the second part immediately after the shell broke and (iv) the distance between the point of firing and the point at which the second part of the shell hits the ground. [I.I T. '70)

[Ans. (i) 500 m/s horizontally (ii) 0 m/s both horizontally and vertically (iii) 1000 m/s along horizontal and 0 m/s along vertical (iv) 3000 m]

55. A block of wood of mass .9 kg is dropped from the top of a tower 30 m high. As it passes by, a person shoots a bullet at it. The bullet of mass .1 kg strikes the block horizontally with a velocity of 100 m/s and gets embedded in it when the block has

fallen a distance of 10 m from the top Neglecting air resistance, (i) calculate the horizontal and the vertical components of the bullet-block combination just after collision and (ii) find the horizontal displacement when the block hits the ground.

[I.I.T. '68]

[Ans. (i) 10 m/s, 12.6 m/s (ii) 11.09 m]

56. Particles, P and Q, of masses 02 kg and .04 kg respectively are simultaneously projected from the points A and B on the ground. The initial velocities of P and Q make angles of 45° and 135° respectively with the horizontal AB. Each particle has an initial speed of 49 m/s. The separation AB is 245 m. Both the particles travel in the same vertical plane and undergo a collision After collision, P retraces its path. Determine the position of Q when it hits the ground. [I I.T. '82]

[Ans. At 122 5 m from A or B i.e. at the mid-point of AB, which is vertically below the highest point of both the trajectories]

57. A body running at a uniform speed of 3 m/s throws up a ball vertically relative to himself and catches it at a place 6 m from the point where he threw it up. With what velocity relative to himself does he throw the ball ? [Ans. 9.8 m/s]

58 A gun is fired at an elevation of $\tan^{-1}\frac{1}{20}$ towards a person at the same horizontal plane as the gun. If the shot and the sound of the gun reach him at the same instant, find the range, the velocity of sound being 336 m/s. (Take $g=10$ m/s^2).

[Ans 1128 96 m]

59. A fountain jet projects stream in all directions with a velocity of 5 m/s from a point 2 m above the centre of the circular basin. What must be the diameter of the basin to catch all the water ? [Ans. 8.18 m]

60. A ball is thrown from the ground level to strike a vertical wall at a distance x away at a height y above the ground when the ball is travelling horizontally. Show that the velocity of projection is

$$\left[g \left(2y+\frac{x^2}{2y} \right)\right]^{\frac{1}{2}}. \qquad \text{[C.U. '66]}$$

61. For a trajectory, show that the focus of the path lies above, on or below the horizontal line through the point of projection,

according as the angle of projection is greater than, equal to, or less than $\frac{\pi}{4}$.

62 A particle which is projected with a velocity u has a range R on a horizontal plane Show that the time of flight, T, and the maximum height, H, can be obtained from the equations

$$g^2 T^4 - 4T^2 u^2 + 4R^2 = 0$$

and $$16gH^2 - 8Hu^2 + gR^2 = 0.$$

63. A vertical rod, PQ, subtends an angle θ at a point O in the same horizontal plane as the foot of the rod. Two balls are projected at the same instant from O, in the directions making angles α and β with the horizontal so that the former strikes the top of the rod at the same moment that the latter strikes the bottom. Prove that $\tan\alpha - \tan\beta = \tan\theta$.

64. A particle is projected from the base of a hill whose shape is that of a right circular cone with axis vertical. The projectile grazes the vertex and strikes the hill again at a point on the base. If α be the semi-vertical angle of the cone, h, its height, V, the initial velocity of the projectile and θ, the angle of projection measured from the horizontal, show that

$$\tan\theta = 2\cot\alpha$$

and $$V^2 = gh\left(2 + \frac{1}{2}\tan^2\alpha\right).$$

65. A particle, being projected with a velocity u at an angle θ with the horizontal, just grazes the edges of a cube (of side a) which stands on a horizontal plane. Show that
$a^2 g^2 = 4u^2 \cos^2\theta\,(u^2 \sin^2\theta - 2ag)$.

66. A fort and a ship are both armed with guns which can fire with a muzzle velocity of $2\,gk$ and the guns in the fort are at a height h above the guns in the ship. If d_1 and d_2 are the greatest horizontal ranges at which the fort and the ship can respectively engage, prove that

$$\frac{d_1}{d_2} = \sqrt{\frac{k+h}{k-h}}.$$

67. Two heavy particles are projected at elevations α and β in the same vertical plane at the same instant with equal velocities from two fixed points, and meet. Show that $\alpha + \beta = \text{constant}$.

68. A fort is situated on the top of a hill at a height h above the sea-level. Prove that the greatest horizontal distance from which a shot can be fired at the fort form a ship in the sea is $\sqrt{2\,k(k-h)}$, it is being given that the initial velocity of the shot is $\sqrt{2gk}$.

69 A body is projected so that on its upward path it passes through a point x m horizontally and y m vertically from the point of projection If R m is the range on the horizontal plane through the point of projection, show that the angle of elevation of the projection is $\tan^{-1}\left(\frac{y}{x}\cdot\frac{R}{R-x}\right)$

70. A boy can throw a ball vertically upwards to a height H m. Show that he cannot clear a wall, h m high, at a distance d m from him unless

$$2\,H \geqslant h+\sqrt{h^2+d^2}$$

71 A tower of height h stands on a horizontal ground, and a particle is projected with speed u from the top so as to hit the ground at the greatest distance from the foot of the tower. Prove that this distance is

$$\frac{u}{g}\sqrt{u^2+2gh}$$

72. If a projectile be at a point $P\,(x,\,y)$ of its path at time t and possesses a velocity V making an angle α with the horizontal, prove that (i) $\tan\alpha=\frac{V_0\sin\theta-gt}{V_0\cos\theta}$

and (ii) $v^2=V_0^2-2\,gy$, where V_0 and θ are the initial velocity and the angle of projection respectively.

If φ be the change in direction of motion in the above problem i.e, if $\varphi=\theta-\alpha$, show that

(iii) $\frac{V_0}{\cos(\theta-\varphi)}=\frac{gt}{\sin\varphi}=\frac{V}{\cos\theta}$ and

(iv) $\tan\varphi=\frac{gt\,\cos\theta}{V_0-gt\sin\theta}$

73. A particle is projected with an initial velocity u. If the greatest height attained by the particle be H, prove that the range R on the horizontal plane through the point of projection is

$$R=4\sqrt{H\left(\frac{u^2}{2g}-H\right)}$$

[C U. ' 40]

74. The line joining P and Q is inclined at an angle α to the horizontal. Show that the least velocity required to shoot from P to Q is to the least velocity required to shoot from Q to P as

$$\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right):\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right).$$

75. Two particles are projected from a point A at a height h above a horizontal plane with equal speeds u, but in opposite directions. Show that, if $u^2>2gh$, the greatest possible distance between the points at which the particles strike the plane is $\frac{u^2}{g}+2h$. What is the greatest possible distance between the points when $u^2<2gh$?

76. A particle P is projected at an angle α to the horizon with velocity V and is subsequently met by a second particle Q, which is let fall from rest from the directrix of the path of P at the instant of projection of P. Show that the distance of the point of projection of P from the straight line described by Q is $\frac{V^2\cot\alpha}{2g}$.

77. A particle projected with a velocity u strikes at right angles to a plane through the point of projection inclined at an angle β to the horizon. Show that the time of flight is

$$\frac{2u}{g\sqrt{1+3\sin^2\beta}}$$

and that the height of the point struck above the point of projection is

$$\frac{\sin^2\beta}{1+3\sin^2\beta}.$$

78. A and B are two points at a distance d apart and at heights h_1, h_2 above a given horizontal plane. Prove that the minimum velocity with which a particle must be projected from the plane so as to pass through A and B is

$$\sqrt{g(h_1+h_2+d)}\ .$$

79. An aeroplane flying with a constant velocity v at a constant height h passes directly over a gun. When the elevation of the

aeroplane above the horizontal plane is θ as seen from the gun, the gun is fired point blank at it Show that the shot hits the aeroplane if $2\,(V\cos\theta - v)\, v \tan^2\theta = gh$, where V is the initial velocity of the shot, its path being parabolic. [C.U. ' 32]

80. Two shots are projected from a gun at the top of a hill with the same velocity u at the angles of projection α and β respectively. If the shots strike the horizontal ground through the foot of the hill at the same point, show that the height h of the hill above the plane is given by

$$h = \frac{2u^2\,(1 - \tan\alpha\tan\beta)}{g\,(\tan\alpha + \tan\beta)}$$

[C.U ' 35]

81. Two particles projected with the same velocity from O, a point on an inclined plane, pass through the same point P in the plane, show that if α, β be the angles of elevation, then

$$\alpha + \beta = \frac{\pi}{2} + i\ ,$$

where i is the angle of inclination of the plane.

82. Show that the product of the two possible times of flights from P to Q with a given velocity of projection is $\dfrac{2\,PQ}{g}$.

83. A shell on striking the horizontal ground bursts and scatters its fragments all with speed V. Show that the area within which a man runs the risk of being hit by a fragment is $\dfrac{\pi V^4}{g^2}$.

84. A particle is projected with a velocity V so that the range on a horizontal plane is twice the greatest height attained. Show that the range is $\dfrac{4V^2}{5\,g}$

85. A particle is projected from a point O so as to pass through two given points in the same vertical plane with O, at heights h_1 and h_2 above O, and at horizontal distances d_1 and d_2 from it on the same side. Show that the angle of projection is

$$\tan^{-1}\left[\frac{d_1^2\,h_2 - d_2^2 h_1}{d_1 d_2\,(d_1 - d_2)}\right]$$

86. For a given horizontal range, if α_1, α_2 are any two possible angles of projection and t_1, t_2, the corresponding times of flight, then prove that

$$\frac{t_1^2 - t_2^2}{t_1^2 + t_2^2} = \frac{\sin(\alpha_1 - \alpha_2)}{\sin(\alpha_1 + \alpha_2)}.$$

87. Particles are projected simultaneously with velocities of magnitude V from a given point in different directions in the same vertical plane. Prove that after time t, they all lie on a circle. [C.U. '41, '53]

88. A particle is projected from a point at a height $3h$ above a horizontal plane, the direction of projection making an angle α with the horizon. Show that if the greatest height above the point of projection is h, the horizontal distance travelled before striking the plane is $6\,h \cot \alpha$.

89. If t be the time of describing any portion PQ of the parabolic path of a projectile, show that t varies

$$(\tan \alpha - \tan \beta),$$

where α and β are the angles which the tangents at P and Q make with the horizon.

90. A ball is projected, and a second ball is also projected from the same point and in the same direction with a velocity equal to the vertical component of the velocity of the first ball. Prove that the path of the second passes through the focus of the path of the first.

91. A body is projected at an angle α to the horizontal so as just to clear two walls of equal height a, at a distance $2a$ from each other. Show that the range is equal to $2a \cot \frac{\alpha}{2}$. [C U. '65]

92. A particle is projected with speed u so as to strike at right angles a plane through the point of projection, inclined at 30° to the horizon. Show that the range on this inclined plane is $\frac{4u^2}{7g}$.

93. A shot is fired at an elevation of α at a bomber flying in a horizontal straight line with acceleration f. At the instant of firing, the bomber is directly overhead at a height h and has a velocity v. Prove that no velocity of projection will make the shot hit the bomber unless

$$\tan \alpha \geqslant \frac{\sqrt{f^2h^2 + 2ghv^2} - fh}{v^2}.$$

94. If R_1 and R_2 be the respective maximum ranges of a particle when projected up and down a plane of elevation α_1, prove that

$$\frac{R_1}{R_2}=\frac{1-\sin\alpha}{1+\sin\alpha}.$$

95. A smooth plane is inclined at an angle of 45° to the horizon and from the foot of it a body is projected upwards along the plane, and reaches the top with $\frac{1}{5}$th of its original velocity u. Show that it will strike the ground at a distance $\frac{16u^2}{25g}$ from the point of projection

96. A particle projected under gravity from a point O passes through two points whose co-ordinates relative to horizontal and vertical axes are (a, a) and $(2a, a)$ Find the angle of projection and show that v, the velocity of projection is given by

$$v^2=\frac{13g\,a}{4}.\qquad\left[\text{Ans. }\tan^{-1}\frac{3}{2}\right]$$

97. If several particles are projected in the same vertical plane from the same point with the same velocity, show that the foci of all the parabolic paths lie on a circle.

98. A ball is projected at an angle α to the horizontal up a plane which passes through the point of projection and is of elevation β Show that it strikes the plane
(i) horizontally, if $\tan\alpha=2\tan\beta$ and
(ii) normally, if $\tan\alpha=2\tan\beta+\cot\alpha$.

99. From the top of a tower, two bodies are projected with the velocity u at different angles α and β of elevation and they meet the horizon at the same place. Show that the distance of the point, where they strike, from the foot of the tower is

$\frac{2u^2}{g(\tan\alpha+\tan\beta)}$ and the height of the tower is

$$\frac{2\,u^2\,(1-\tan\alpha\tan\beta)}{g\,(\tan\alpha+\tan\beta)^2}.$$

100. A particle is projected with velocity V and angle of elevation a from a point O. Prove that if V is given, the particle may be made to pass through any point P inside the parabola

$$4g^2\,x^2=V^2\,(V^2-4\,gy).$$

by a suitable choice of α. Given that P is at a height h above the level of O, and that $OP = d$, prove that the least velocity of projection with which a particle can be projected from O so as to pass through P is $\sqrt{g(h+d)}$.

101. Particles are projected from a given point in the same vertical plane at the same elevation. Show that the locus of (i) the vertices and (ii) the foci of the parabolas which they describe is each a straight line.

102. What is the angle of projection of a particle so that the focus of the path may be as much below the point of projection as the vertex is above?

$$\left[\text{Ans. } \sin^{-1}\frac{1}{\sqrt{3}}\right]$$

103. If two particles are simultaneously projected from the same point in the same vertical plane, prove that the line joining them moves parallel to itself.

104. Two particles are projected simultaneously form O in different directions with the same speed u so as to pass through another point P. If α and β are the anlges of projection, prove that they pass through P at times separated by

$$\frac{2u}{g} \cdot \frac{\sin\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}.$$

105. If the velocity V at any point of the path of a projectile is suddenly diminished by one-half, prove that the focus of the new trajectory is instantaneously shifted nearer to the projectile through the distance $\frac{3V^2}{8g}$.

106. The radii of the front and hind wheels of a carriage are a and b, and c is the distance betwten their axle-trees. A particle of dust driven from the highest point of the hind wheel is observed to alight on the highest point of the front wheel. Show that the velocity of the carriage is

$$\sqrt{\frac{(c+a-b)(c-a+b)}{4(b-a)}g}.$$

107. A balloon, rising vertically from the ground with constant acceleration f, was struck by a shell fired at the instant when

the balloon was released If the shell was fired from a gun on the ground at a distance a from the point of release of the balloon, show that the least possible initial speed of the shell was $\sqrt{a(f+g)}$ and find the angle of elevation of the gun for this speed of projection [Ans 45°]

108. If R is the greatest horizontal range possible for a particle projected with a given velocity, without any restriction, prove that when the vertical height is restricted not to exceed $h\left(\frac{1}{4}<R\right)$, the greatest range attainable is $2\sqrt{2h(R-2h)}$

109 A plane is flying horizontally with constant velocity v m/s. A shot, fired from a gun G on the ground when the plane is at A, hits the plane when it is at B, t s later. If AG makes an angle α with the horizontal, prove that $v=\frac{1}{2}gt\cot\alpha$.

110 If the greatest height attained by a rifle bullet is $\frac{1}{n}$th the range on a horizontal plane through the point of projection, show that unless $n=4$, it is possible to increase the range by altering the angle of projection

APPENDIX

SOME MULTIPLE-CHOICE OBJECTIVE TYPE QUESTIONS

In most of the competitive examinations, the multiple-choice objective type questions are asked. In Mechanics, these types of questions are hardly available. A few such questions are given below from each topic of this book for the convenience of students.

In each of the following questions, there are four possible answers marked (A), (B), (C) and (D). Only one of these answers is correct. The letter that has been given against each correct answer should be written in bold letter in the bracket provided in the right-hand margin.

The value of g, the acceleration due to gravity, should be taken as 9 8 m/s^2 in S I system in all problems where it occurs, unless otherwise stated.

I. Topic . Fundamental Concepts, Average Speeds and Velocities and Composition and Resolution of Velocities

1. The statement which is never true is that all

(A) vector quantities have directions

(B) vector quantities have magnitudes

(C) scalar quantities have directions

(D) scalar quantities have magnitudes ( )

2. When we say that a moving point has a velocity v, we mean that it would undergo a displacement of

(A) v units of length in v units of time

(B) unit length in v units of time

(C) v units of length in unit time

3. A velocity can be resolved into

(A) only two components

(B) only three components

(C) only four components

(D) any number of components ( )

4. A man travelled from Calcutta to Delhi by a train. The train moved with

(A) a uniform speed

(B) a uniform acceleration

(C) a uniform speed and a uniform acceleration

(D) none of these ( )

5 A body moves in a circular path of radius 42 m It moves 66 m in 3 s and completes the path in 12 s nearly. Therefore, the body moves with

(A) a uniform velocity

(B) a uniform speed

(C) a uniform acceleration

(D) a variable acceleration ( )

6. If the final displacement of a particle moving with a constant speed is zero, then it must have moved along a

(A) straight line

(B) circle

(C) parabola

(D) hyperbola ( )

7. A particle moves with a velocity of 5 m/s from O to A The component of the velocity along AO will be

(A) 5 m/s

(B) —5 m/s

(C) 0 m/s

8. AB is a line segment such that $AB = a$ m. A particle starts from A to B with a uniform velocity of u m/s and comes back to A again in time ts. Then the displacement of the particle is

   (A) a m

   (B) $2a$ m

   (C) ut m

   (D) none of the above ( )

9. A man moves 3m along east, then 8 m along north and then again 3m along east. The displacement of the man is

   (A) 6 m

   (B) 14 m

   (C) 10 m

   (D) 8 m ( )

10. A particle moves in a straight line with a velocity of 3 m/s. After $\frac{1}{3}$ s it has an additional velocity of 3 m/s at right angles to its original direction of motion. The displacement of the particle 1 s after it has the additional velocity is

    (A) 5 m

    (B) $\sqrt{13}$ m

    (C) $1 + 3\sqrt{2}$ m

    (D) none of the above ( )

11. A cyclist starts from A to B with a speed of 30 m/s and returns to A from B with a speed of 20 m/s. The average speed of the cyclist will be

    (A) 24 m/s

    (B) 25 m/s

    (C) 0 m/s

    (D) none of the above ( )

12. If u km/h be the velocity of a body at A and v km/h be the velocity at B. x km be the distance from A to B and t hours be the time

taken by the body to go from A to B, then the average velocity will be

(A) $\frac{u+v}{2}$ km/h

(B) $\frac{x}{t}$ km/h

(C) $\frac{v-u}{6}$ km/h

(D) none of the above ( )

13. A man walks 3 km towards east at the rate of 5 km/h and then walks 4 km towards north at the rate of 3 km/h. Then for the whole journey the man's average

(A) speed is $3\frac{18}{29}$ km/h

(B) speed is $\frac{29}{30}$ km/h

(C) velocity is $3\frac{18}{29}$ km/h north of east

(D) velocity is $\frac{29}{30}$ km/h north of east ( )

14. An aeroplane flies 100 km due west then 1000 km due north. Then it flies back to the starting place by the shortest route. Its speed is 400 km/h throughout the journey Then the average velocity of the aeroplane over the entire trip is

(A) 400 km/h

(B) 200 km/h

(C) 100 km/h

(D) 0 km/h ( )

15. A car covers the first half of the distance between two places at a speed of 40 km/h and the other half at 60 km/h. Then the average speed of the car is

(A) 50 km/h

(B) 48 km/h

(C) 100 km/h

(D) 55 km/h

16. A boy goes a distance x m with a speed v_1 m/s and returns the same by v_2 m/s. His average speed is

(A) $\frac{v_1 + v_2}{2}$ m/s

(B) $\frac{2x}{v_1 + v_2}$ m/s

(C) 0 m/s

(D) $\frac{2}{\frac{1}{v_1} + \frac{1}{v_2}}$ m/s ( )

17. If a particle moving along a straight line describes first p m of its journey with a uniform velocity of u m/s and the remaining q m of its journey with a uniform velocity of v m/s, then the magnitude of its average velocity is

(A) $\frac{pu + qv}{p + q}$ m/s

(B) $\frac{pv + qu}{p + q}$ m/s

(C) $\frac{(p + q)\,uv}{pv + qu}$ m/s

(D) $\frac{u + v}{p + q}$ m/s ( )

18. A particle is moving in a straight line AB and its displacement from A after t s is given by the relation $S = 0.25\,t + 0.375\,t^2$. The average velocity during the fourth second is

(A) 3.25 m/s

(B) 2.875 m/s

(C) 3.5 m/s

(D) none of the above ( )

19. A boy is going 6 m due east and then 8 m due north in 10 s with a

(A) $\frac{14}{10}$ m/s

(B) $\frac{48}{10}$ m/s

(C) $\frac{10}{10}$ m/s

(D) $\frac{2}{10}$ m/s ( )

20. A particle has a velocity v in a direction, v is a resolved part of v in a direction making an angle α with v. Then the resolved part of v' along the direction of v is

(A) $v \sin \alpha$

(B) $v \cos \alpha \sin \alpha$

(C) $v \cos^2 \alpha$

(D) v ( )

21. The components of a given velocity V along two directions making angles α and β with the direction of the velocity will be

(A) $\frac{V \cos \beta}{\sin (\alpha + \beta)}$ and $\frac{V \sin \beta}{\sin (\alpha + \beta)}$

(B) $\frac{V \sin \alpha}{\sin (\alpha + \beta)}$ and $\frac{V \cos \alpha}{\sin (\alpha + \beta)}$

(C) $\frac{V \sin \beta}{\cos (\alpha + \beta)}$ and $\frac{V \sin \alpha}{\sin (\alpha + \beta)}$

(D) $\frac{V \sin \beta}{\sin (\alpha + \beta)}$ and $\frac{V \sin \alpha}{\sin (\alpha + \beta)}$ ( )

22. If a velocity of 100 m/s resolved into two equal velocities at an angle of 60° to each other, then the magnitude of the equal velocity is

(A) 50 m/s

(B) $\frac{100\sqrt{3}}{3}$ m/s

(C) $50\sqrt{3}$ m/s

23. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is

(A) $\frac{1}{8}$ m/s

(B) $\frac{\sqrt{3}-1}{4}$ m/s

(C) $\frac{\sqrt{6}-\sqrt{2}}{8}$ m/s

(D) none of the above ( )

24. A particle possessing simultaneously three velocities 7 m/s, 8 m/s and 8 m/s is at rest. The angle between the greatest and the smallest velocities

(A) is $\frac{\pi}{3}$ radian

(B) is $\frac{\pi}{2}$ radian

(C) is $\frac{2\pi}{3}$ radian

(D) can not be determined from the given data ( )

25. An aeroplane travels 100 km to the north and then 200 km to the east. In order to reach its destination in a straight flight, the aeroplane should have headed to the east of north at an angle which

(A) is less than 30°

(B) lies between 30° and 45°

(C) lies between 45° and 60°

(D) more than 60° ( )

26. A bird can fly in still air at 20km/h. When wind is blowing straight from the north at 10 km/h, the direction in which the bird should aim in order that it may fly from east to west is

(A) 30° west of north

(B) 60° north of west

(C) 30° north of west

27. If two velocities each equal to u act at a point at an angle θ, their resultant velocity is

(A) $2u \cos \frac{\theta}{2}$

(B) $2u \cos \theta$

(C) $2u \sin \frac{\theta}{2}$

(D) $2u \sin \theta$ ( )

28. A particle can, in no circumstance, be at rest if it possesses the velocities given by

(A) 20 cm/s, 10 cm/s, 8 cm/s

(B) 7 cm/s, 8 cm/s, 13 cm/s

(C) 3 cm/s, 7 cm/s, 8 cm/s

(D) 3 cm/s, 4 cm/s, 7 cm/s ( )

29. A particle has velocities 3 m/s, 7 m/s and 13 m/s inclined at an angle of 120° to one another. The resultant velocity is

(A) $3\sqrt{3}$ m/s

(B) $2\sqrt{19}$ m/s

(C) $4\sqrt{21}$ m/s

(D) $\sqrt{349}$ m/s ( )

30. Two velocities include an angle θ when their resultant is v. If the angle be $\frac{\pi}{3} - \theta$, the resultant is $\frac{v}{2}$ and if the angle be $\frac{\pi}{3} + \theta$, then the resultant is $\frac{v}{3}$. Then the value of θ will be

(A) $\sin^{-1} \frac{5\sqrt{3}}{59}$

(B) $\cos^{-1} \frac{5\sqrt{3}}{59}$

(C) $\tan^{-1} \frac{59\sqrt{3}}{5}$

(D) $\cot^{-1} 59\sqrt{3}$

31. A particle possesses simultaneously three velocities u, v and w making angles α, β and γ with a given straight line. Then the resultant velocity is

(A) $(u^2 + v^2 + w^2)^{\frac{1}{2}}$

(B) $[u^2 + v^2 + w^2 + uv \cos(\alpha - \beta) + vw \cos(\beta - \gamma)$

$+ wu \cos(\alpha - \gamma)]^{\frac{}{2}}$

(C) $u + v + w$

(D) $[u^2 + v^2 + w^2 + 2\,uv \cos(\alpha - \beta) + 2\,vw \cos(\beta - \gamma)$

$+ 2\,wu \cos(\alpha - \gamma)]^{\frac{1}{2}}$ ( )

32. A point possesses simultaneously two velocities along perpendicular directions, the resultant velocity being 17 km/h. If each velocity is decreased by 3 km/h, then the resultant velocity becomes 13 km/h. The two velocities are

(A) 15 km/h and 10 km/h

(B) 15 km/h and 8 km/h

(C) 10 km/h and 8 km/h

(D) none of the above ( )

33. At any instant a particle possesses two velocities of magnitude 3 units and 4 units. The maximum possible resultant is

(A) 1 unit

(B) 5 units

(C) 7 units

(D) none of the above ( )

34. A particle has three velocities 1 m/s, 2 m/s and 3 m/s which are in directions parallel to the sides of an equilateral triangle, taken in order. Then the magnitude of the resultant of the velocities is

(A) $\sqrt{3}$ m/s

(B) $2\sqrt{3}$ m/s

(C) $\sqrt{19}$ m/s

35. If the resultant of two equal velocities be equal in magnitude to one of the velocities, then the angle between the directions of the given velocities will be

(A) $\tan^{-1}\frac{1}{2}$

(B) $\tan^{-1}(-\sqrt{3})$

(C) $\tan^{-1}(\sqrt{3})$

(D) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ ( )

36 A particle possesses two velocities which are at right angles. If the angle between the velocities is gradually increased upto 180°, then the magnitude of the resultant velocity will

(A) increase only

(B) decrease only

(C) remain unchanged

(D) increase first and then decrease ( )

37. Two inextensible ropes are tied to a heavy object which is initially at rest on the ground. Holding the ends of the ropes tightly, two boys, *A* and *B*, are running perpendicularly with velocities 1 m/s and $\sqrt{3}$ m/s respectively. Then the object will move

(A) towards the boy *A*

(B) towards the boy *B*

(C) in a direction making an angle of 30° with the direction of the boy *B*

(D) in a direction making an angle of 30° with the direction of the boy *A* ( )

38. A particle possesses *n* simultaneous coplanar velocities of magnitudes $V_1, V_2, \ldots V_n$ making angles $\theta_1, \theta_2, \theta_3 \ldots \theta_n$ with a fixed line. The

(A) $\sum_{r=1}^{n} V_r \cos \theta_r = 0, \quad \sum_{r=1}^{n} V_r \sin \theta_r \neq 0$

(B) $\sum_{r=1}^{n} V_r \cos \theta_r = 0, \quad \sum_{r=1}^{n} V_r \sin \theta_r = 0$

(C) $\sum_{r=1}^{n} V_r \cos \theta_r$ and $\sum_{r=1}^{n} V_r \sin \theta_r$ act perpendicularly

(D) $\sum_{r=1}^{n} V_r \cos \theta_r \neq 0, \quad \sum_{r=1}^{n} V_r \sin \theta_r = 0$ ( )

39. If a moving point possesses velocities represented by $\lambda \overrightarrow{OA}$ and $\mu \overrightarrow{OB}$, they are equivalent to a velocity

(A) $(\lambda + \mu) \overrightarrow{OP}$, where P is on AB such that $\lambda AP = \mu PB$

(B) $(\lambda - \mu) \overrightarrow{OP}$, where P is on AB such that $\lambda AP = \mu PB$

(C) $(\lambda + \mu) \overrightarrow{OP}$, where P is on AB such that $\mu AP = \lambda PB$

(D) $(\lambda - \mu) \overrightarrow{OP}$, where P is on AB such that $\mu AP = \lambda PB$ ( )

40. Let PABC be a square. The resultant of velocities represented by $\overrightarrow{PA}$, $2 \overrightarrow{PB}$, $2 \overrightarrow{CP}$ is

(A) $3 \overrightarrow{BC}$

(B) $3 \overrightarrow{PA}$

(C) $3 \overrightarrow{AP}$

(D) $3 \overrightarrow{AB}$ ( )

41. A point has two velocities u and v inclined at such an angle that their resultant is equal to u; if u is doubled, the new resultant is inclined with the velocity v at an angle of

(A) 30°

(B) 60°

(C) 90°

42. A road is running parallel to a railway track and the distance between the road and the track is m. A man sitting in the train sees a horse running on the road in the direction of a train. He observes that the horse is always in line with a tree which is situated on the side of the road opposite to the railway track at a distance of n from the road. Then the ratio of the velocity of the train to that of horse is

(A) $1 + \frac{m}{n}$

(B) $1 + \frac{n}{m}$

(C) $1 - \frac{m}{n}$

(D) $1 - \frac{n}{m}$ ( )

43. A man of height h m in front of a light post of height H m moves with a velocity of V m/s away from the light post. Then the velocity of the farthest end of the shadow of the man due to light on the light post is

(A) $\frac{HV}{H-h}$ m/s

(B) $\frac{hv}{h-H}$ m/s

(C) $\frac{HV}{h-H}$ m/s

(D) $\frac{h}{H-h}$ m/s ( )

44. A point has equal velocities in two given directions. If one of these velocities being halved, the angle which the new resultant makes with the other is half the angle between the original resultant and this velocity. The angle between the original velocities is then

(A) 60°

(B) 90°

(C) 120°

(D) none of th

45. A particle P has simultaneously three velocities represented by $\overrightarrow{PA}$, $\overrightarrow{PB}$ and $\overrightarrow{PC}$, where A, B, C, are three non-collinear points. If the resultant velocity of P is zero, then P must be situated at

(A) the circumcentre of $\triangle$ABC

(B) the centroid of $\triangle$ABC

(C) the in-centre of $\triangle$ABC

(D) the othorcentre of $\triangle$ABC ( )

46. If a moving point possesses simultaneously velocities represented in magnitude and direction by the two sides AB and BC of a triangle ABC, taken in order, they are equivalent to a velocity represented in magnitude and direction by

(A) the median of the triangle through the vertex B

(B) AC

(C) CA

(D) none of the above ( )

47. A particle is moving with constant velocity parallel to the y-axis and a velocity proportional to y parallel to the x-axis. The particle will describe

(A) a straight line

(B) a circle

(C) a parabola

(D) an ellipse

48. At (x, y) in the xy-plane, the components of velocities of a moveable particle parallel to x and y axes are $-k^2y$ and $-k^2x$ respectively. If the particle starts from $(a\ 0)$, then the locus of the particle will be

(A) a straight line

(B) a circle

(C) a parabola

49. A man can row his boat with velocity u in still water. A river is flowing with velocity v. In order to reach a point on the other side of the river just opposite to the point of start, the man should row his boat in a direction making an angle with the upstream, given by

(A) $\sin^{-1}\left(\frac{v}{u}\right)$

(B) $\cos^{-1}\left(\frac{v}{u}\right)$

(C) $\sin^{-1}\left(-\frac{v}{u}\right)$

(D) $\cos^{-1}\left(-\frac{u}{v}\right)$ ( )

50. A man runs directly across a flowing river in time t_1 and rows an equal distance down the stream in time t_2. If u be the speed of the man in still water and v, that of the stream, then the value of $\frac{t_1}{t_2}$ is

(A) $\sqrt{\frac{u-v}{u+v}}$

(B) $\sqrt{\frac{u+v}{u-v}}$

(C) $\frac{u-v}{u+v}$

(D) $\frac{u+v}{u-v}$ ( )

51. A boat takes m minutes to go 1 km down stream and n minutes to go 1 km upstream. The speed of the current and that of the boat are respectively

(A) $30\left(\frac{n-m}{n+m}\right)$ km/h, $\frac{n+m}{mn}$ km/h

(B) $\frac{n-m}{mn}$ km/h, $60\left(\frac{n+m}{mn}\right)$ km/h

(C) $60\left(\frac{n+m}{mn}\right)$ km/h, $60\left(\frac{n-m}{mn}\right)$ km/h

(D) $30\left(\frac{n-m}{mn}\right)$ km/h, $30\left(\frac{n+m}{}\right)$ km/h

52 A boat is rowed with a velocity of 4 km/h straight across a river flowing at 2 km/h. If the river is 600 m broad, the boat will be carried down the stream through a distance

(A) 0 m

(B) 300 m

(C) 200 $\sqrt{3}$ m

(D) 400 m ( )

53 A boat capable of moving in still water with a speed of 5 km/h crosses a river, 1 km broad, flowing with a valocity of 3 km/h. Then the time of crossing the river by the shortest route is

(A) $\frac{1}{5}$ h

(B) $\frac{1}{3}$ h

(C) $\frac{1}{4}$ h

(D) $\frac{1}{8}$ h ( )

54. A man wishes to swim across a river in least time. If the velocity of flow of the river and the velocity with which the man can swim in still water be equal, the path which he would actually describe is a straight line inclined at an angle

(A) 30° to the direction of flow

(B) 45° to the direction of flow

(C) 30° to the direction opposite to that of flow

(D) 45° to the direction opposite to that of flow ( )

55. A man rows directly across a river in time t and rows an equal distance down the stream in time T. The ratio of his speed in still water to that of the speed of the stream is

(A) $\frac{t - T}{t + T}$

(B) $\frac{t^2 + T^2}{t^2 - T^2}$

(C) $\frac{t^2 - T^2}{t^2 + T^2}$

$t + T$

56. Two straight railway lines meet at right angles. A train starts from the junction along one line, and at the same instant another train starts towards the junction from a station on the other line, which is at a distance s from the junction and they move with the same uniform speed. Then the least distance between the trains during their journey is

(A) $\frac{s}{2}$.

(B) $\frac{(\sqrt{2}+1)s}{2\sqrt{2}}$

(C) $\frac{s}{\sqrt{2}}$

(D) none of the above ( )

57. A swimmer can swim 1 km along a stream in x h and he takes y h to swim the same distance against the stream. The velocity of the swimmer is

(A) $\frac{x - y}{2xy}$ km/h

(B) $\frac{x + y}{2xy}$ km/h

(C) 0 km/h

(D) $\frac{2}{x + y}$ km/h ( )

58. A steamer can cross a river of width s m directly in t s in still water, but in flowing river the steamer takes t' s to cross the same river directly. The velocity of the river will be

(A) $s\sqrt{\frac{1}{t'^2} - \frac{1}{t^2}}$ m/s

(B) $s\sqrt{\frac{1}{t^2} - \frac{1}{t'^2}}$ m/s

(C) $s\sqrt{\frac{1}{t^2} + \frac{1}{t'^2}}$ m/s

(D) none of the above ( )

ANSWERS

1. (C) 2. (C) 3. (D) 4. (D) 5. (B) 6. (B) 7. (B) 8. (D) 9 (C) 10. (B)
11. (A) 12. (B) 13. (A) 14. (D) 15. (B) 16 (D) 17. (C) 18. (B) 19. (C)
20. (C) 21 (D) 22. (B) 23. (C) 24. (C) 25. (D) 26. (C) 27. (A) 28. (A)
29. (B) 30. (D) 31. (D) 32. (B) 33. (C) 34. (A) 35. (B) 36. (B) 37 (C)
38. (B) 39. (A) 40. (B) 41. (C) 42. (A) 43. (A) 44. (C) 45. (B) 46. (B)
47. (C) 48. (D) 49 (B) 50. (B) 51. (D) 52. (B) 53 (C) 54. (B) 55. (B)
56. (C) 57. (B) 58. (B)

II. Topic : Relative Velocity

1. The relative velocity of a body *A* with respect to another body *B* is obtained by

   (A) compounding the actual velocities of *A* and *B*

   (B) compounding the actual velocity of *A* and that of *B* in opposite directions

   (C) compounding the actual velocity of *B* and that of *A* in opposite directions

   (D) none of the above methods ( )

2. Two points *A* and *B* are on a straight line. *A* is moving with uniform velocity along *AB*. The relative velocity of *A* with respect to to *B* is 2 *v*. Then

   (A) *B* moves along *AB* with uniform velocity *v*

   (B) *B* moves along *AB* with uniform velocity 2 *v*

   (C) *B* moves along *BA* with uniform velocity *v*

   (D) *B* moves along *BA* with uniform velocity 3 *v* ( )

3. Two persons begin to run along the circumference of a circle in the same direction from the same point with uniform speeds of 3 m/s and 5 m/s respectively. If the length of the circumference be 90 m, then they will again meet together after a time

   (A) 30 s

   (B) 18 s

   (C) 11.25 s

   (D) 45 s ( )

4. The hero of a stunt film jumps on the top of a moving train of length 200 m at its rear end and runs along the top towards the engine. If a person on the ground finds that the speed of the hero relative to the ground is 40 km/h and the speed of the train relative to the ground is 30 km/h, the time taken by the hero to reach the engine is

(A) 18 s

(B) $\frac{4320}{7}$ s

(C) 72 s

(D) $\frac{72}{7}$ s ( )

5. Two trains of length 140 m and 180 m respectively are observed to pass each other in opposite directions on parallel lines completely in 8 s. If the velocity of the longer train be three times that of the other, then the velocity of the longer train is

(A) 10 m/s

(B) 30 m/s

(C) 20 m/s

(D) 60 m/s ( )

6. A particle A is moving with a velocity 3 m/s with respect to a particle B moving with 1 m/s in the same direction. Another particle C is moving in the direction opposite to B with velocity 2 m/s. The relative velocity of C with respect to A is

(A) 2 m/s

(B) 6 m/s

(C) 4 m/s

(D) 3 m/s ( )

7. Two trains whose lengths are respectively $2l$ and $3l$ are moving in opposite directions with velocities v and $4\ v$ respectively

(A) $\frac{l}{3v}$

(B) $\frac{l}{v}$

(C) $\frac{l}{5v}$

(D) $\frac{5l}{3v}$ ( )

8. A train of length 60 m travelling at 40 km/h overtakes another of length 50 m travelling in the same direction at 30 km/h. The time taken by the first train to overtake the second is

(A) $\frac{9}{2}$ s

(B) $\frac{90}{5}$ s

(C) $\frac{108}{5}$ s

(D) $\frac{198}{5}$ s ( )

9. A train A is moving with a speed of 40 km/h in a straight direction pointing towards east. Another train B is also moving due east on an adjoining parallel track with a speed of 60 km/h. A person sitting in A will observe that B moves with a speed of

(A) 20 km/h due west

(B) 20 km/h due east

(C) 100 km/h due west

(D) 100 km/h due east ( )

10. Let $\vec{V}_{AB}$ represent the relative velocity of A with respect to B If $\vec{V}_{AB}$ and $\vec{V}_{CB}$ be given, then $\vec{V}_{CA}$ is obtained by compounding

(A) $\vec{V}_{AB}$ and $\vec{V}_{CB}$

(B) $\vec{V}_{BA}$ and $\vec{V}_{CB}$

(C) $\vec{V}_{AB}$ and $\vec{V}_{BC}$

11. One caterpillar and an ant start simultaneously from the same point and move along two straight lines, one with uniform velocity u and the other from rest with uniform acceleration f. If α be the angle between the two lines, then the least relative velocity is

(A) $u \cos \alpha$

(B) $u \sin \alpha$

(C) $u \tan \alpha$

(D) $u \sec \alpha$ ( )

12. A boat X is sailing with a uniform velocity u due north and another Y with a uniform velocity v due west. If X were initially at a distance x west of Y, then their shortest distance will be

(A) $\frac{xv}{\sqrt{u^2 + v^2}}$

(B) $\frac{xu}{\sqrt{u^2 + v^2}}$

(C) $\frac{xuv}{\sqrt{u^2 + v^2}}$ ( )

(D) none of the above

13. Two bodies are moving with different velocities in different directions. The relative velocity of one with respect to the other is maximum when the angle between the two is

(A) 0°

(B) 90°

(C) 180°

(D) none of the above ( )

14. Two particles move with speed u and v ($v > u$) in the same direction along the circumference of a circle. Their relative velocity is least in magnitude when

(A) they meet

(B) they are at diametrically opposite positions

(C) the diameters through them are at right angles

15. Two particles move along the circumference of a circle in opposite dirrctions with velocities 2 m/s and 4 m/s respectively. Their relative velocity will be greatest when the angle between the two velocities is

    (A) 0°

    (B) 90°

    (C) 180°

    (D) none of the above ( )

16 A school boy holding an umbrella runs with a velocity equal to that of the rain falling vertically. In order to protect him best, he should hold the umbrella at an angle α with the vertical, where α is equal to

    (A) 0°

    (B) 90°

    (C) 45°

    (D) none of the above ( )

17. A man is walking at a uniform velocity of 5 km/h and the rain-drops fall vertically with the same velocity. Then the rain drops will hit the man

    (A) on the forehead at an angle $\frac{\pi}{4}$

    (B) on the head vertically

    (C) on the back at an angle $\frac{\pi}{4}$

    (D) horizontally ( )

18. A ship S_1 is steaming due east at the rate of 12 km/h and another ship S_2 due north at 16 km/h. An observer sitting in ship S_1 finds S_2 moving at an angle

    (A) $\sin^{-1}(3/5)$ east of north

    (B) $\sin^{-1}(3/5)$ west of north

    (C) $\sin^{-1}(4/5)$ east of north

19. A boat X is moving due east and another boat Y is moving due south. The person on boat X will see that the boat Y is moving due

(A) south

(B) south-west

(C) south-east

(D) north ( )

20. Rain is falling vertically downwards with velocity of 60 m/s. But to a person in a running train, it appears to fall at an angle of 30° to the vertical. Then the velocity of the train is

(A) 60 m/s

(B) $60\sqrt{3}$ m/s

(C) $20\sqrt{3}$ m/s

(D) $30\sqrt{3}$ m/s ( )

21. Rain is falling with a velocity of 60 m/s at an angle 30° to the vertical. But to a car moving with a velocity u, it appears to fall vertically. Then u is equal to

(A) 60 m/s

(B) 30 m/s

(C) 120 m/s

(D) $20\sqrt{3}$ m/s ( )

22. A person travelling eastwards finds that the wind seems to blow directly from north ; on doubling his speed it seems to come from north-east. If he trebles his speed, the wind will appear to blow from a direction making an angle

(A) $\tan^{-1}\left[\frac{1}{2}\right]$ to the north

(B) $\tan^{-1}\left[\frac{1}{2}\right]$ to the east

(C) $\cot^{-1}\left[\frac{1}{2}\right]$ to the north

(D) $\cot^{-1}\left[\frac{1}{2}\right]$ to the east ( )

23. Three particles at A, B and C are moving along the sides AB, BC and CA respectively of an equilateral triangle ABC with velocities each of magnitude u relative to a stationary observer. Then the magnitude of the velocities of the particles at A, B and C relative to

(A) u, 0 and $\sqrt{3}\,u$

(B) $\sqrt{3}\,u$, 0 and $\sqrt{3}\,u$

(C) $\sqrt{3}\,u$, 0 and u

(D) u, $\sqrt{3}\,u$ and 0 ( )

24. Two particles, A and B are moving with velocities V_A and V_B respectively and their directions are inclined with each other. The tangent of the angle that the relative velocity of B with respect to A makes with the direction of V_A is

(A) $\dfrac{V_A \sin \alpha}{V_B - V_A \cos \alpha}$

(B) $\dfrac{V_B \sin \alpha}{V_A - V_B \cos \alpha}$

(C) $\dfrac{V_A \sin \alpha}{V_A + V_B \cos \alpha}$

(D) $\dfrac{V_B \sin \alpha}{V_B \cos \alpha - V_A}$ ( )

25. A man is standing at a distance of 10 m from a mirror. He is advancing to the mirror at a uniform speed of 2 m/s. His image formed in the mirror is also advancing with respect to the man with uniform speed of

(A) 2 m/s

(B) 4 m/s

(C) 5 m/s

(D) 0 m/s

26 The relative velocity of A with respect to B is given by

$$V = \sqrt{a^2 - b^2 \cos \alpha}\ ,$$

where a and b are dependent on the velocities of A and B and α is the angle between their directions of motion. If u, v be the actual velocities of A and B, then a and b must satisfy the relation

(A) $u^4 - v^4 = (a^2 - b^2)^2$

(B) $a^2 + b^2 = (u - v)^2$

(C) $a^4 - b^4 = (u^2 - v^2)^2$

ANSWERS

1. (B) 2. (C) 3. (D) 4. (C) 5. (B) 6. (B) 7. (B) 8. (D) 9. (B) 10. (B) 11. (B) 12. (B) 13. (C) 14. (A) 15. (C) 16. (C) 17. (A) 18. (B) 19. (B) 20. (C) 21. (B) 22. (B) 23. (B) 24. (D) 25. (B) 26. (C).

III. Topic : Motion in a Straight Line with Acceleration

1. A Particle is moving along a straight line. It describes a distance x during time t, where $x = 7 + 24t - 2t^2$. Then the particle moves with a

(A) uniform acceleration

(B) uniform retardation

(C) variable acceleration

(D) uniform velocity ( )

2. A particle moves over 16 m, 40 m and 70 m respectively in the first, the second and the fourth seconds of its motion along a straight line. Then the particle is moving with a

(A) uniform acceleration

(B) uniform retardation

(C) uniform velocity

(D) non-uniform acceleration

3. For a uniformly accelerated motion of a particle in a straight line, the space described in successive seconds form

(A) an A.P

(B) a G.P.

(C) a H.P.

(D) none of these

4. At an instant, the relation between the velocity of a particle and its distance from a fixed point is given by $v = 10 + \frac{s}{15}$. Then the acceleration will be

   (A) 10 times the velocity

   (B) 15 times the velocity

   (C) $\frac{1}{15}$ times the velocity

   (D) none of these ( )

5 The velocity v and the distance y from a fixed point of a particle moving on a straight line through the fixed point is given by the relation $v^2 = A + By$. Then the acceleration of the particle at that instant is

   (A) $\frac{B}{2}$

   (B) B

   (C) $2B$

   (D) indeterminate ( )

6. A particle moves in a straight line so that its distance measured from a fixed point on the line at time t is given by $x = t^3 - 6t^2 - 15t$. The time interval during which its velocity is negative, but its acceleration is positive is given by

   (A) $2 < t < 5$

   (B) $5 < t < 10$

   (C) $0 < t < 2$

   (D) none of the above ( )

7. A particle is moving according to the law $v^2 = 2(x \sin x + \cos x)$, where x is the distance described. Its maximum acceleration is

   (A) equal to 1

   (B) less than 1

   (C) more than 1

8. If the distance x traversed by a particle along a straight line at any instant t is given by $x = A \cos nt + B \sin nt$, then the acceleration of the particle is

(A) $-n^2 A$

(B) $-n^2 B$

(C) $n^2 x$

(D) none of the above ( )

9. If a particle moves along a straight line so that its distance s at any time t from a fixed point in the line is given by $s^2 = at^2+2\,bt+c$, a, b, c being constants, then the acceleration varies as

(A) $\frac{1}{s^2}$

(B) s^2

(C) $\frac{1}{s^3}$

(D) none of the above ( )

10. If the law of motion of a body moving along a straight line is $x = \frac{1}{2}\,vt$, then the acceleration is

(A) a constant

(B) proportional to v

(C) proportional to x^2

(D) none of the above ( )

11. The velocity of a particle moving in a straight line is given by $v = \mu \sqrt{\frac{c-x}{x}}$, where c and μ are constants and x is the displacement of the particle from a certain fixed point on the line of motion. The acceleration of the particle is given by

(A) $-\frac{c\mu^2}{2x^2}$

(B) $-\frac{c\mu^2}{2vx^2}$

(C) $\frac{c\mu^2}{2x^2}$

12. The velocity v m/s of a bullet is reduced to 0 when the bullet penetrates 3 m into a target. If it penetrates 1 m, the velocity becomes u m/s. Therefore, $u : v$ will be equal to

(A) $\sqrt{2} : \sqrt{3}$

(B) $2 : \sqrt{3}$

(B) $\sqrt{2} : 1$

(D) none of the obove ( )

13. A bullet hits a target of width 20 m with a velocity of 70 m/s directly and penetrates 10 m in $\frac{1}{6}$ s It will come just out of the target in a total time of

(A) $\frac{1}{3}$ s

(B) $\frac{1}{2}$ s

(C) $\frac{2}{3}$ s

(D) $\frac{3}{2}$ s ( )

14. A bullet travelling with a uniform velocity penetrates a wooden block of widhth 100 cm. After crossing the wooden block, it again continues to move with some other uniform velocity. Let t_1 s and t_2 s be respectively the times taken by the bullet to cross the wooden block and a distance of 100 cm in air after crossing the block. Then $\frac{t_1}{t_2}$ is

(A) equal to 1

(B) less than 1

(C) more than 1

(D) not possible to work out with present data ( )

15. A bullet fired into a plank loses $\frac{2}{7}$th of its velocity after penetrating the plank. It then moves with the same velocity and strikes a similar second plank which it just passes through. Then the ratio of the thicknesses of the planks is

(A) 24 : 49

(B) 25 : 49

(C) 24 : 25

(D) 4 : 49 ( )

16. A body moving along a straight line has a velocity of 15 m/s at a certain moment. Its velocity is 45 m/s after 10s. If the velocity changes uniformly, then the space described by the body in 10 s is

(A) 150 m

(B) 225 m

(C) 300 m

(D) 375 m ( )

17. A particle is moving along a straight line with a uniform acceleration and s m is the space described by the particle in time t s and s' m during the next t' s. Then the acceleration of the particle will be

(A) $\dfrac{2\left(\frac{s'}{t'}-\frac{s}{t}\right)}{t+t'}$

(B) $\dfrac{2\left(\frac{s}{t}-\frac{s'}{t'}\right)}{t-t'}$

(C) $\dfrac{2\left(\frac{s}{t}+\frac{s'}{t'}\right)}{t+t'}$

(D) none of the above ( )

18. A particle moving with a uniform retardation in a straight line passes through A, B, C, where $AB = BC = d$, taking twice as long to pass from B to C as from A to B. Then the ratio of the speeds of the particle at A, B, C is

(A) 7 : 5 : 1

(B) 5 : 7 : 1

(C) 5 : 1 : 7

(D) 1 : 7 : 5

19. If s and s' are the spaces traversed by a point moving with a uniform acceleration in a straight line in times t and t' respectively $(t' > t)$, reckoned from the same instant, then the acceleration of the point is

(A) $\dfrac{2(s't - st')}{tt'(t' - t)}$

(B) $\dfrac{s't - st'}{tt'(t' - t)}$

(C) $\dfrac{2(s't + st')}{tt'(t' - t)}$

(D) $\dfrac{2(s't - st')}{tt'(t' + t)}$ ( )

20. A particle moves along a straight line, starting with a velocity u and an acceleration f. When the velocity has increased to $5u$, the acceleration is reversed in direction, its magnitude being unalterd. The magnitude of the velocity of the particle when it has returned to its starting point is

(A) u

(B) $5u$

(C) $7u$

(D) $9u$ ( )

21. A point moves in a straight line from A to B with a uniform acceleration. If P be the position at half time and u and v, the initial and final velocities respectively, then $AP : PB$ is equal to

(A) $\dfrac{u + v}{3v - u}$

(B) $\dfrac{3u + v}{u + 3v}$

(C) $\dfrac{3u + v}{5v - u}$

22. A particle starting with a velocity u moves in a straight line with uniform acceleration f. If it travels a distance s in time t and acquires a velocity v in that time, then s can not be equal to

(A) $\frac{1}{2}(u+v)\,t$

(B) $ut+\frac{1}{2}f\,t^2$

(C) $\frac{v^2-u^2}{2f}$

(D) $u+\frac{1}{2}f(2t-1)$ ( )

23. A particle starts with initial velocity u m/s to move in a straight line and passes over the $\frac{1}{3}$ of a given distance with uniform acceleration f_1 m/s^2 and over the first half of the rest with uniform acceleration f_2 m/s^2 and finally over the rest with uniform acceleration f_3 m/s^2. The final velocity is the same as if the whole distance were traversed with uniform acceleration f m/s^2. Then f is equal to

(A) $f_1+f_2+f_3$

(B) $\frac{1}{2}(f_1+f_2+f_3)$

(C) $\frac{1}{3}(f_1+f_2+f_3)$

(D) $3(f_1+f_2+f_3)$ ( )

24. A particle starts moving from the position of rest under constant acceleration. It travels a distance x in the first two seconds and a distance y in the next two seconds. Then

(A) $y=x$

(B) $y=2x$

(C) $y=3x$

25. A particle is moving with constant acceleration. Its initial velocity is u and final velocity is v. If v be its velocity at half time and v'' be its velocity at half way, then $v''^2 - v'^2$ is equal to

(A) $-\frac{1}{4}(u-v)^2$

(B) 0

(C) $\frac{1}{4}(u-v)^2$

(D) none of the above ( )

26. Two horses, Jack and Henry, move along a straight track *AB*. Starting from *A*, Jack moves with velocity u and acceleration f and Henry with velocity u' and acceleration f'. If they both have the same velocity at the middle point of *AB*, then *AB* is equal to

(A) $\frac{v'^2 - u'^2}{f - f'}$

(B) $\frac{u'^2 - v'^2}{f + f'}$

(C) $\frac{u'^2 - u^2}{f - f'}$

(D) $\frac{v'^2 - u'^2}{f + f'}$ ( )

27 A body moves for three seconds with a constant acceleration and describes a distance of 81 m. The acceleration then ceases to exist and during the next three seconds, it describes a distance of 72 m. The initial velocity and the acceleration of the body are respectively

(A) 30 m/s and 2 m/s^2

(B) 30 m/s and $-$ 2 m/s^2

(C) 45 m/s and 2 m/s^2

(D) 35 m/s and $-$ 2 m/s^2

28. A body starting from rest and moving with uniform acceleration describes a distance of 245.1 m in 10th second. The acceleration of the body is

(A) 4.9 m/s^2

(B) 25 8 m/s^2

(C) 25.5 m/s^2

29. A train travels a distance s starting from rest and finishing its journey at rest. The velocity increases uniformly till it reaches a maximum value v and then decreases uniformly. The total time of motion is

(A) $\frac{s}{v}$

(B) $\frac{s}{2v}$

(C) $\frac{2s}{v}$

(D) none of the above ( )

30. Two cars A and B are at a distance of 80 m from each other. B starts with a velocity u and an acceleration f moving away from A. It will be possible for A to overtake B moving with the same acceleration f in 8 s if A starts moving with initial velocity greater than that of B by

(A) 8 m/s

(B) 10 m/s

(C) 4 m/s

(D) 80 m/s ( )

31. A particle starting with a initial velocity and a uniform acceleration covers a distance of x m in t th second and a distance of y m in $(t+n)$ th second. Its acceleration will be

(A) $\frac{y-x}{n}$ m/s^2

(B) $\frac{x-y}{n}$ m/s^2

(C) $\frac{x+y}{n}$ m/s^2

(D) $-\frac{x+y}{n}$ m/s^2 ( )

32. The average velocity of a particle moving along a straight line with a uniform acceleration during any interval of time is not equal to

(A) the velocity at the middle of the interval

(B) the mean of the initial and final velocities

(C) the quotient of space by time

(D) the velocity attained if the particle moves with twice the given acceleration ( )

33. If the time-displacement graph of a particle is parallel to the time axis, then the particle has

(A) a uniform velocity

(B) a uniform acceleration

(C) a uniform retardation

(D) a zero velocity ( )

34. The space-time curve for a uniformly accelerated motion of a particle in a straight line is a part of a

(A) straight line

(B) parabola

(C) ellipse

(D) none of the above ( )

35. Three accelerations are represented in magnitude and direction by the sides of a right-angled triangle of sides 3 units, 4 units and 5 units, taken in order. Then the magnitude of their resultant is

(A) 5 units

(B) $\frac{5}{7}$ units

(C) $\frac{5}{12}$ units

36. Three accelerations which are represented in magnitude and direction by the sides of a triangle ABC, taken in order, where $CA < AB < BC$, are impressed on a particle at O. Then the particle will be moving with a uniform

(A) acceleration parallel to AC

(B) acceleration parallel to AB

(C) acceleration parallel to BC

(D) velocity ( )

37. Two particles move from the same point along the same line, one with a uniform velocity of u m/s and the other with a uniform acceleration of f m/s^2 and no initial velocity. If the first particle is x m behind the second particle, then the particles will never meet when

(A) $u^2 < \sqrt{2fx}$

(B) $u^2 > \sqrt{2fx}$

(C) $u^2 > 2fx$

(D) $u^2 < 2fx$ ( )

38. A tortoise and a hare start simultaneously from the same point and move along two straight lines, one with uniform velocity u and the other from rest with uniform acceleration f. If α be the angle between the lines, then their relative velocity is least after a time

(A) $\dfrac{u \sin \alpha}{f}$

(B) $\dfrac{u \sec \alpha}{f}$

(C) $\dfrac{u \operatorname{cosec} \alpha}{f}$

(D) $\dfrac{u \cos \alpha}{}$ ( )

ANSWERS

1. (B) 2. (D) 3. (A) 4. (C) 5. (A) 6. (A) 7. (C) 8. (D) 9. (C) 10. (A) 11. (A) 12. (A) 13 (B) 14. (B) 15. (C) 16. (C) 17. (A) 18. (A) 19. (A) 20. (C) 21. (B) 22. (D) 23. (C) 24. (C) 25. (C) 26. (C) 27. (B) 28. (B) 29. (C) 30. (B) 31. (A) 32. (D) 33. (D) 34. (B) 35. (D) 36. (A) 37. (D) 38. (D)

IV. Topic : Motion in a Straight line under Gravity

1. The magnitude of the acceleration of a stone thrown upwards is

   (A) always greater than that of a stone thrown downwards

   (B) always the same as that of a stone thrown downwards

   (C) always smaller than that of a stone thrown downwards

   (D) sometimes greater and sometimes smaller than that of a stone thrown downwards ( )

2. A particle is projected vertically upwards with a velocity u. It will return to the point of projection with a velocity equal to

   (A) $2u$

   (B) u

   (C) $\frac{u}{2}$

   (D) none of the above ( )

3. A body is projected vertically upwards with a given velocity u and t_1 is the time taken by the particle to move from a point on the path to the greatest height and t_2 is the time taken by the particle to move from the greatest height back to the same point. Then

   (A) $t_1 = 2t_2$

   (B) $2t_1 = t_2$

   (C) $t_1 = t_2$

4. A particle is projected vertically upwards and takes 50 s to reach the top. It will return to the point of prejection after a total time of

(A) 100 s

(B) 150 s

(C) 200 s

(D) 50 s ( )

5. Two objects of the same mass but of different radii and densities are thrown vertically upwards in air at two different places Calcutta and Paris at the same time with the same velocity. Then their times of flight will be different, because their

(A) densities are different

(B) radii are different

(C) shapes are different

(D) weights are different ( )

6. A body is released from the surface of the earth into a well with an initial velocity of 10 m/s. It reaches the bottom of the well in 10 s. Then the depth of the well is

(A) 490 m

(B) 590 m

(C) — 390 m

(D) none of the above ( )

7. A ball was thrown upwards and was caught exactly after 20 s. The maximum height it has reached is

(A) 49 m

(B) 490 m

(C) 980 m

8. Two bodies are projected vertically upwards with velocities u and v respectively. If the maximum heights attained by them are in the ratio 25 : 36, then $u : v$ will be

   (A) 6 : 5

   (B) 36 : 25

   (C) 5 : 6

   (D) none of the above ( )

9. A particle is projected vertically upwards with a given velocity u Its velocity at a height $\frac{h}{2}$, where h is the greatest height attained by the particle, is

   (A) $\frac{u}{2}$

   (B) $\frac{u}{\sqrt{2}}$

   (C) $\sqrt{2}u$

   (D) $2u$ ( )

10. A body is falling freely and the acceleration due to gravity is taken as 10 m/s^2. The distances described by the body in the first and the last seconds are in the ratio of 1 . 15. The velocity with which it strikes the ground is

    (A) 20 m/s

    (B) 80 m/s

    (C) 320 m/s

    (D) indeterminate ( )

11. A particle is projected vertically upwards with a velocity of 30 m/s from the ground. If $g = 10$ m/s^2, it will remain above 40 m for

    (A) 4 s

    (B) 1 s

    (C) 2 s

12. A particle is thrown vertically upwards with a velecity which is sufficient to raise it to a height of 4.9 m. The velocity of the particle when it just reaches the ground is

(A) 9.8 m/s

(B) 0 m/s

(C) 4.9 m/s

(D) indeterminate from the given data ( )

13. A body is projected vertically upwards with a velocity u m/s It passes through a point in its path with a velocity v m/s. It will again pass through the same point after

(A) $\frac{2v}{g}$ s

(B) $\frac{2u}{g}$ s

(C) $\frac{v^2}{2g}$ s

(D) $\frac{u^2}{2g}$ s

14. A particle is dropped from a height of 60 m. After falling two-thirds of this height, it passes through a particle thrown upwards from the ground at the instant the first was dropped. Then the velocity of projection of the second particle is

(A) 7 m/s

(B) $\sqrt{784}$ m/s

(C) 21 m/s

(D) $\sqrt{1176}$ m/s ( )

15. A particle is dropped from a height h After falling through $\frac{2}{3}h$, it meets with a particle thrown vertically upwards from the ground at the instant when the first is dropped. The latter rises to a maximum height equal to

(A) $\frac{3}{8}h$

(B) $\frac{5}{8}h$

(C) $\frac{7}{8}h$

(D) h ( )

16. A particle is projected vertically upwards with a velocity of u m/s and after t s, another particle is projected upwards from the same point and with the same velocity. The first particle will meet with the second after a total time of

(A) $\left(\frac{t}{2}-\frac{u}{g}\right)$ s

(B) $\left(\frac{t}{2}+\frac{u}{g}\right)$ s

(C) $\left(\frac{u}{g}-\frac{t}{2}\right)$ s

(D) $\left(t+\sqrt{\frac{2ut}{g}-t^2}\right)$ s ( )

17. A stone falling from rest from the top of a vertical tower has descended a m when another is let fall from rest from a point x m below the top. If they reach the ground together, then the height of the tower is

(A) $\frac{(a+x)^2}{4a}$ m

(B) $\frac{(a-x)^2}{4a}$ m

(C) $\frac{a+x}{4a}$ m

$(a-x)^2$

18. A body is dropped from the top of a tower of height h m and at the same instant another body is projected from the foot of the tower in the same vertical line with a velocity which is sufficient to take the body to the top of the tower. They meet after

(A) $\frac{h}{\sqrt{2gh}}$ s at a distance of $\frac{h}{4}$ m from the top

(B) $\frac{h}{\sqrt{2gh}}$ s at a distance of $\frac{h}{2}$ m from the top

(C) $\frac{h}{\sqrt{2gh}}$ s at a distance of $\frac{h}{4}$ m from the ground

(D) $\frac{h}{4}$ s at a distance of $\frac{h}{\sqrt{2gh}}$ m from the top ( )

19 A body is let fall. It describes a certain distance and this distance is divided into n equal parts. The ratio of the time of describing the first part to that of describing the last is

(A) $1 : (\sqrt{n} - \sqrt{n-1})$

(B) $1 : (\sqrt{n} + \sqrt{n-1})$

(C) $1 . (\sqrt{n+1} - \sqrt{n})$

(D) $1 : (\sqrt{n+1} + \sqrt{n})$ ( )

20. If a particle falls freely under geravity from rest, then the ratio of its velocity at half time to its velocity half-way is

(A) $1 : 1$

(B) $\sqrt{2} : 1$

(C) $1 : \sqrt{2}$

(D) none of the above ( )

21. If H_1 and H_2 are the heights to which a body can be projected with a given initial velocity at two places on the earth's surface, then the

(A) $\sqrt{H_1} : \sqrt{H_2}$

(B) $\sqrt{H_2} : \sqrt{H_1}$

(C) $H_1 . H_2$

(D) $H_2 : H_1$ ( )

22. If two bodies are thrown vertically upwards with initial velocities in the ratio of 2 : 5, then the maximum heights attained by them will be in the ratio of

(A) 2 : 5

(B) 5 : 2

(C) 4 : 25

(D) $\sqrt{2} : \sqrt{5}$ ( )

23 A particle takes t seconds less and acquires a velocity of s m/s more at one place where the acceleration due to gravity is g_1 than at another place where the acceleration due to gravity is g_2 in falling from rest through the same distance. The gravities g_1 and g_2 at the two places are related by

(A) $g_1 + g_2 = \frac{s^2}{t^2}$

(B) $g_1 - g_2 = \frac{t^2}{s^2}$

(C) $g_1 g_2 = \frac{s^2}{t^2}$

(D) $\frac{g_1}{g_2} = \frac{s^2}{t^2}$ ( )

24. If a particle takes t s less and acquires a velocity v m/s more at one place where the acceleration due to gravity is g_1 than at another place where the same is g_2, in falling freely through the same distance, then the ratio $v : t$ is equal to

(A) $\sqrt{g_1 g_2} : 1$

(B) $g_2 : g_1$

(C) $g_1 g_2 : 1$

25. A particle is projected vertically upwards. If t_1 and t_2 be the times at which it passes through a height h above the point of projection during the ascent and descent respectively, then $t_1 t_2$ is equal to

(A) $\frac{h}{g}$

(B) $\frac{h}{2g}$

(C) $\frac{2h}{g}$

(D) $\frac{2g}{h}$ ( )

26. A particle is projected vertically upwards with a velocity u m/s. If t_1, t_2 are the times at which it passes a point at a height h above the point of prejection in ascending and descending respectively, then the height h and the initial velocity u will respectively be

(A) t_1t_2g m and $\frac{2}{g}(t_1+t_2)$ m/s

(B) $\frac{2}{g}(t_1+t_2)$ m and t_1t_2g m/

(C) $\frac{1}{2}t_1t_2g$ m and $\frac{g}{2}(t_1+t_2)$ m/s

(D) $\frac{g}{2}(t_1+t_2)$ m and $\frac{1}{2}t_1t_2g$ m/s ( )

27. A particle thrown vertically upwards takes a time t to rise to a height h and a further time t' to reach the ground again. Then h is equal to

(A) $g(t+t')$

(B) gtt'

(C) $\frac{1}{2}gtt'$

28. A body is dropped from rest at a height of u^2 m above the ground. When it is just at a height of x m above the ground, the velocity of the body will will be

(A) $\sqrt{u^2 - 2gx}$

(B) $\sqrt{u^2 + 2gx}$

(C) $\sqrt{2gx}$

(D) none of the above ( )

29. From the roof of a building h m high, a stone is let fall and it strikes the ground in 2 s, If the velocity of the stone on reaching the ground is v m/s, then h is equal to

(A) $2v$ m

(B) $\frac{v}{2}$ m

(C) 19 6 m

(D) $\frac{1}{2}vg$ m

30. Two particles are dropped from certain height at an interval of 1 s. After t s of dropping the first particle, the two particles will be at a distance of

(A) $\frac{1}{2}g(t-1)$ apart

(B) $\frac{1}{2}g(2t-1)$ apart

(C) $g(t-1)$ apart

(D) $\frac{1}{2}g(2t+1)$ apart ( )

31. From three positions A, B, C on the same vertical line, three particles are respectively let fall from A, projected with velocity u vertically upwards from B and thrown vertically downwards with velocity v so that they reach the ground at the same time. If h_1, h_2

(A) $h_1 > h_2 > h_3$

(B) $h_3 > h_2 > h_1$

(C) $h_2 > h_1 > h_3$

(D) $h_3 > h_1 > h_2$ ( )

32. A particle is projected vertically upwards with a velocity u. After t s the particle acquires a velocity v at a height h and t' is the subsequent time to reach the ground. Then h is *not* equal to

(A) $ut - \frac{1}{2} gt^2$

(B) $vt' + \frac{1}{2} gt'^2$

(C) $-vt' + \frac{1}{2} gt'^2$

(D) $\frac{1}{2} gtt'$ ( )

33. A body is dropped fom a height h above the ground from an aeroplane moving horizontally with a velocity u and t is the time taken by the aeroplane to reach the ground. Then the velocity of the aeroplane on reaching the ground is given by

(A) $u + gt$

(B) $u - gt$

(C) $\sqrt{2gh}$

(D) none of the above ( )

34. A particle falls freely from the top of a tower. If during the last second of its motion, it falls $\frac{5}{9}$th of the whole height, then the height of the tower is equal to

(A) 1.764 m

(B) 44.1 m

(C) 24.5 m

35. From a balloon at a height of 24.5 m above the ground, a stone is dropped. If the balloon be ascending with a uniform velocity of 24 5 m/s, the time taken by the stone to reach the ground can be obtained from the equation

(A) $t^2 - 5t + 5 = 0$

(B) $t^2 - 5t - 5 = 0$

(C) $t^2 + 5t - 5 = 0$

(D) $t^2 + 5t + 5 = 0$ ( )

36. *AB* is the vertical diameter of a circle whose plane is vertical, and *PQ*, a diameter is inclined at an angle θ to *AB*. If the time of sliding down *PQ* be twice that of sliding down *AB*, then the value of θ is

(A) $\sin^{-1} \frac{1}{4}$

(B) $\cos^{-1} \frac{1}{4}$

(C) $-\sin^{-1} \frac{1}{4}$

(D) $\cos^{-1} \frac{1}{2}$ ( )

37. Two smooth wires are tied on a vertical circle with their one end at the highest point of the circle and the other ends are tied on other points of the circle. If the length of one wire is twice that of the other, then the ratio of times of desend of a body along these wires is

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) indeterminate ( )

38. The time taken by a particle falling freely along any chord which makes an angle θ in radian with the vertical diameter of a verti-

(A) directly proportional to θ

(B) inversely proportional to θ

(C) inversely proportional to $\sqrt{\theta}$

(D) independent of the angle θ ( )

39. Four points P, Q, R, S are on a vertical circle, taken in the anti-clockwise order, P and R being on the highest and lowest positions of the circle respectively. Two particles start from rest from P and move along the smooth chords PQ and PS respectively. As soon as the particles reach the positions Q and S, a third and a fourth particles start from rest from Q and S respectively along the smooth chords QR and SR. If the third and the fourth particles reach R at the same time, then

(A) $PQ < PS$

(B) $PS < PQ$

(C) $PQRS$ forms a square

(D) none of the above conditions is always necessary ( )

40. If from the same height h, two particles be allowed to slide down different inclinations α and 2α respectively and v_1, v_2 be their velocities on reaching the ground, then

(A) $v_1 = 2\,v_2$

(B) $v_2 = 2\,v_1$

(C) $v_1 = v_2$

(D) $v_1 = 2\,v_2 \cos\alpha$ ( )

41. A body sliding on a smooth inclined plane requires 4 s to reach the bottom, starting from rest at the top. The time taken by it to cover one-fourth of the distance starting from rest at the top is

(A) 1 s

(B) 2 s

(C) 4 s

42. Let t_1 be the time taken by a body to fall down a smooth inclined plane of inclination 45° to the horizon and t_2 be the time of falling of the body down the vertical height of the plane Then

(A) $t_2 = t_1 \sqrt{2}$

(B) $t_1 = t_2 \sqrt{2}$

(C) $t_1 = t_2$

(D) none of the above is true ( )

43. A body is allowed to slide down a smooth inclined plane of inclination α to the horizon from rest. If the initial velocity of projection is $\sqrt{5}$ m/s, the height of the plane is 1 m and the acceleration due to gravity is 10 m/s², then the velocity on reaching the ground is

(A) $\sqrt{5 + 20 \sin \alpha}$ m/s

(B) $5 \sin \alpha$ m/s

(C) $5\sqrt{\sin \alpha}$ m/s

(D) 5 m/s ( )

44. A body is allowed to slide down a smooth inclined plane of inclination α to the horizontal plane from a vertical height h and another falls vertically downwards from rest from the same height. If v_1 and v_2 are the velocities of the respective bodies on reaching the ground, then

(A) $v_1 = v_2 \sin \alpha$

(B) $v_2 = v_1 \sin \alpha$

(C) $v_1 = v_2$

(D) none of the above is true ( )

45. A body sliding down a smooth inclined plane is observed to pass [illegible] distances each equal to d in consecutive intervals, t_1

(A) $\cos^{-1}\left[\dfrac{2d\,(t_1 - t_2)}{g t_1 t_2\,(t_1 + t_2)}\right]$

(B) $\tan^{-1}\left[\dfrac{2d\,(t_1 - t_2)}{g t_1 t_2\,(t_1 + t_2)}\right]$

(C) $\sin^{-1}\left[\dfrac{2d\,(t_1 - t_2)}{g t_1 t_2\,(t_1 + t_2)}\right]$

(D) $\sec^{-1}\left[\dfrac{2d\,(t_1 - t_2)}{g t_1 t_2\,(t_1 + t_2)}\right]$ ( )

46. A block is released from rest at the top of a frictionless inclined plane 16 m long. It reaches the bottom 4 s later. A second block is projected up the plane from the bottom at the instant the first block is released in such a way that the second block returns to the bottom simultaneously with the first block. Then the initial velocity of the second block is

(A) 2 m/s

(B) 8 m/s

(C) 4 m/s

(D) none of the above ( )

47. A ball was moving down a smooth inclined plane of inclination 30° starting from rest from the top of the plane. After it had moved for 2 s, the inclination of the plane is changed to 90°. The ball struck the ground in a total time of 4 s. If $g = 10\ \text{m/s}^2$, then the length of the plane is

(A) 80 m

(B) 50 m

(C) 40 m

(D) none of the above ( )

48. A man on a lift ascending with acceleration $f\ \text{m/s}^2$ throws a ball vertically upwards with velocity v relative to the lift and catches it

(A) $g - f = \frac{2v}{t}$

(B) $f - g = \frac{2v}{t}$

(C) $f + g = \frac{2v}{t}$

(D) $-(f + g) = \frac{2v}{t}$ ( )

49. Consider a car moving with a constant scceleration a_x along a straight horizontal path. Consider a ball of mass m inside the car, the ball being acted upon by the pull of gravity only Then the acceleration experienced by the ball relative to the car is $(a_x^2 + g^2)^{\frac{1}{2}}$ acting

(A) upwards at an angle $\theta = \tan^{-1}\frac{g}{a_x}$ with the vertical

(B) upwards at an angle $\theta = \tan^{-1}\frac{a_x}{g}$ with the vertical

(C) downwards at an angle $\theta = \tan^{-1}\frac{g}{a_x}$ with the vertical

(D) downwards at an angle $\theta = \tan^{-1}\frac{a_x}{g}$ with the vertical ( )

50. It is estimated that g is increased by the resistance of the air to $g(1 + f)$ in upward motion and reduced to $g(1 - f)$ in downward motion. The time a stone is in the air when projected upwards with velocity u is

(A) $\frac{u}{g}\left(\frac{1}{1+f} + \frac{1}{1-f}\right)$

(B) $\frac{u}{g}\left(\frac{1}{1-f} + \frac{1}{1+f}\right)$

(C) $\frac{u}{g}\left(\frac{1}{1+f} + \frac{1}{\sqrt{1-f^2}}\right)$

(D) $\frac{u}{g}\left(\frac{1}{\sqrt{1+f}} + \frac{1}{1-f}\right)$

51. A body is thrown up in a lift with a velocity u relative to the lift and the time of flight is found to be t. If the lift is moving up with an acceleration f, then the value of f is

(A) $\frac{u + tg}{t}$

(B) $\frac{2u - tg}{t}$

(C) $\frac{2u + tg}{t}$

(D) none of the above ( )

ANSWERS

1. (B) 2. (B) 3. (C) 4. (A) 5. (D) 6. (B) 7. (B) 8. (C) 9. (B)
10. (B) 11 (C) 12. (A) 13. (A) 14. (C) 15. (A) 16. (B) 17. (A)
18. (A) 19. (A) 20. (C) 21. (D) 22. (C) 23. (C) 24. (A) 25 (C)
26. (C) 27. (C) 28. (A) 29. (C) 30. (B) 31. (D) 32. (B) 33. (C)
34. (B) 35. (B) 36. (B) 37. (A) 38. (D) 39. (D) 40. (C) 41. (B)
42. (B) 43. (D) 44. (C) 45. (C) 46. (C) 47. (B) 48. (C) 49. (D)
50. (C) 51. (B)

V. Topic : Laws of Motion

1. A particle of mass m acted upon by a force P in a given direction measured from a fixed origin undergoes a displacement x at time t. The velocity at that instant is v. The expression which will not give the force is

(A) $m\frac{dv}{dt}$

(B) $mv\frac{dv}{dx}$

(C) $m\frac{d^2x}{dt^2}$

(D) $mv\frac{dv}{dt}$

2. 1 Newton is the magnitude of a force that produces an acceleration f upon a body of man m where

   (A) $m = 1$ gm and $f = 1$ cm/s^2

   (B) $m = 1$ ton and $f = 1$ km/s^2

   (C) $m = 1$ kg and $f = 1$ cm/s^2

   (D) none of the above statements is true ( )

3. Newton's second law of motion cannot be always expressed as

   (A) $P=m\dfrac{dv}{dt}$

   (B) $P=m\dfrac{dv}{dx}$

   (C) $P=mg$

   (D) $P=m\dfrac{d^2x}{dt^2}$

4. The action and reaction forces referred to in Newton's third law of motion

   (A) act upon the same body

   (B) act upon different bodies

   (C) need not be equal in magnitude but must have the same line of action

   (D) must be equal in magnitude but need not have the same line of action ( )

5. The pressure of a body on a horizontal plane cannot be equal to the weight of the body when the plane is

   (A) at rest

   (B) ascending with a uniform velocity

   (C) descending with a uniform velocity

6. A man was gliding in a balloon. To find the motion of the balloon, he suspended a 5 kg weight from a spring balance and to his surprise the balance reads 5.5 kg. So his correct conclusion was that the balloon was

   (A) ascending with an acceleration

   (B) descending with an acceleration

   (C) gliding horizontally

   (D) falling under gravity ( )

7. A lift is going up with an acceleration. A man in the lift throws a a ball above and catches it back after t s. During this time the displacement of the ball will be

   (A) 0

   (B) greater than the displacement of the lift

   (C) less than the displacement of the lift

   (D) equal to the displacement of the lift ( )

8. A man is standing at the centre of a smooth and horizontal table. He can come out of the table by

   (A) running

   (B) rolling

   (C) throwing something forward from his hand

   (D) none of the above methods ( )

9. A man stands in a lift by taking a spring balance from which a mass of 2 kg is suspended. Now there will be no change in reading in the balance if he is

   (A) ascending with an acceleration of .5 m/s^2

   (B) descending with an acceleration of .5 m/s^2

   (C) ascending with a retardation of .5 m/s^2

   (D) descending with

10. A man weighing m kg is ascending in a lift with an acceleration of a m/s^2. If the chain of the lift breaks during ascent, the thrust of the feet on the lift will be

    (A) $m(g + a)$ N

    (B) $m(g - a)$ N

    (C) 0 N

    (D) none of the above ( )

11. The pressure exerted by a man on the floor of a lift is zero if the lift is

    (A) at rest

    (B) moving downwards with an acceleration g

    (C) moving upwards with a variable acceleration f

    (D) moving with a uniform velocity ( )

12. Consider a particle of mass m inside a freely falling elevator, the particle being acted upon by a force of magnitude F An observer inside the elevator detects that the acceleration of the particle is of magnitude

    (A) $\dfrac{F}{m}$

    (B) $\sqrt{\left(\dfrac{F}{m}\right)^2 + g^2}$

    (C) g

    (D) zero ( )

13. With usual notations, the relation between a force acting on a mass m producing an acceleration f is given by $P = mf$ It is incorrect to say that

    (A) the relation holds good only when the acceleration due to gravity is assumed to be constant

    (B) the relation holds good at any point on the earth's surface

    (C) the relation holds good outside the surface of the earth

14. The pressure of a body resting on a horizontal plane which is moving with a uniform velocity will be

(A) increased when moving upwards

(B) increased when moving downwards

(C) decreased when moving downwards

(D) the same when moving upwards or downwards as that when it is at rest ( )

15. A thief of mass m_1 jumps off the terrace of a building when a heavy suitcase of mass m_2 on his head and falls vertically. Then the pressure of the suitcase on his head while he is falling is

(A) $(m_1 + m_2)\, g$

(B) $(m_2 - m_1)\, g$

(C) 0

(D) none of the above ( )

16. A body of mass m is placed on a horizontal plane and the plane is rising vertically upwards with an acceleration f. Then the pressure R in the plane is given by

(A) $R = m(g + f)$

(B) $R = m(g - f)$

(C) $R = mf$

(D) $R = mg$ ( )

17. A man with a load of 10 kg in his hand is standing on the floor of a lift. If the lift descends with an acceleration of 1 m/s^2, then the pressure of the load on his hand will be

(A) 0 kg

(B) 10 kg

(C) greater than 10 kg

18. A man carrying a heavy load with his hand is descending in a lift with a uniform retardation which is numerically less than g. Then he will feel that the load is

    (A) heavier

    (B) lighter but not weightless

    (C) of the same weight as that when the lift was at rest

    (D) weightless ( )

19. A man carrying a heavy load in his hand is descending in a lift with a uniform acceleration which is numerically less than g Then he will feel that the load is

    (A) heavier

    (B) lighter but not weightless

    (C) of the same weight as that when the lift was at rest

    (D) weightless ( )

20. A man is standing on a platform which is descending with a uniform acceleration f. He will fly in space if

    (A) $g > f$

    (B) $g < f$

    (C) $g = f$

    (D) f is not related to g ( )

21 A force of 10 N is applied to a 4 kg block that is at rest on a perfectly smooth level surface. In 6 s, the body will move through a distance of

    (A) 2.5 km

    (B) 45 km

    (C) 45 m

22. A lift with a man descends with an acceleration f. The man appear lighter by

(A) $(f - g)$ of his actual weight

(B) $(f + g)$ of his actual weight

(C) $\frac{f}{g}$ of his actual weight

(D) fg of his actual weight ( )

23. A book is placed upon another book and then the lower book is dropped from a height h above the ground. During the time of falling, the upper book exerts a force on the lower book which will be

(A) zero

(B) equal to the actual weight of the upper book

(C) less than the actual weight of the upper book

(D) greater than the actual weight of the upper book ( )

24. A piece of stone of mass m rests at a height h above the surface of the ground. It then falls and penetrates into the ground. If it comes to rest after penetrating a distance a into the ground, then the average resistance of the ground is

(A) $\frac{mgh}{a}$

(B) $m\left(\frac{h}{a}+1\right)g$

(C) $m\left(\frac{h}{a}-1\right)g$

(D) none of the above ( )

25. A force of 5 N when acted on masses m_1 and m_2 successively produces accelerations of 8 m/s^2 and 24 m/s^2 respectively. The two masses are now fastened together and the same force is applied. The

(A) greater than 24

(B) greater than 8 but less than 24

(C) 5

(D) 6 ( )

26. A body acted upon by a uniform force moves through 1 m in 10 s from rest. The ratio of the force to the weight of the body is

(A) 10 : 49

(B) 1 : 49000

(C) 1 . 490

(D) none of the above ( )

27. Two equal forces are acting on two different masses M kg and m **kg** separately and produce velocities V m/s and v m/s respectively in a certain interval of time. Then the masses M and m of the bodies are in the ratio

(A) 1 · 1

(B) $v : V$

(C) $V : v$

(D) $2\,V : v$ ( )

28 A thin glass plate can just support a weight of 3 kg. A body is placed on it and the plate is raised with the body on it with a gradually increasing acceleration. It is found that the plate just breaks when the acceleration is 5 m/s^2. If the acceleration due to gravity is 10 m/s^2, then the mass of the body is

(A) 3 kg

(B) greater than 3 kg

(C) $\frac{3}{5}$ kg

29. A body whose true weight is 14 kg appeared to weigh 13 kg when weighed by means of a spring balance in a lift moving downwards. The acceleration of the lift at the time of weighing is

(A) .7 m/s^2

(B) 1.4 m/s^2

(C) $-$ .7 m/s^2

(D) $-$ 1.4 m/s^2 ( )

30. A body of mass $\frac{1}{4}$ kg moves in a straight line so that its distance from the origin at the end of t s is given by the equation $s=5+4 \sin 3t$. Then the maximum force exerted upon it during the motion is

(A) 6 N

(B) 12 N

(C) 9 N

(D) 36 N ( )

31. A cricket ball of mass $\frac{1}{5}$ kg moving with a velocity of 20 m/s is brought to rest by a player in 1 s. The average force applied by the player is

(A) 40 N

(B) 400 N

(C) 20 N

(D) none of the above ( )

32. A thin glass plate can bear the weight of 246 g. A body of certain mass is placed on the plate and it is lifted with an increasing acceleration. When the acceleration is 4 cm/s^2 and if $g = 980$ cm/s^2, then the plate will be at the point of breaking if the mass of the body is just

(A) 244 g

(B) 245 g

(C) 243 g

33. A body of mass 10 kg can be brought to rest by an application of a force within 12 s. Another force can bring the same body moving with double the velocity to rest in 3 s. The ratio of the second force to the first force is

(A) 8 : 1

(B) 1 : 8

(C) 4 : 1

(D) none of the above ( )

34, A balloon of mass m_1 is rising in air with certain acceleration f. If, with the same volume of the balloon, its mass be reduced to m_2, the acceleration becomes three times its original acceleration. Then the ratio $m_1 : m_2$ is

(A) $\frac{f - g}{3f - g}$

(B) $\frac{3f - g}{f - g}$

(C) $\frac{3f + g}{f + g}$

(D) 3 ( )

ANSWERS

1. (D) 2. (D) 3. (C) 4. (B) 5. (D) 6. (A) 7. (D) 8. (C) 9. (D) 10. (C) 11. (B) 12. (A) 13. (A) 14 (D) 15. (C) 16. (A) 17. (D) 18. (A) 19. (B) 20. (B) 21. (C) 22. (C) 23 (A) 24. (B) 25. (D) 26. (C) 27. (B) 28 (D) 29 (A) 30. (C) 31. (A) 32. (B) 33. (A) 34. (C)

VI. Topic : Motion under connected Systems

1. Two particles of masses m_1 and m_2 $(m_1 > m_2)$ are connected by a light inextensible string which passes over a small smooth fixed pulley, Then the statement which is never true is that
   (A) the tension of the string is the same throughout its length
   (B) accelerations of the particles are different due to different masses of the particles
   (C) accelerations of the particles are the same
   (D) displacements of the particles after a particular time are the same ( )

2. Two particles of masses m_1 and m_2 are connected by a light inextensible string. The mass m_2 is placed on a smooth horizontal table, the string passes over a light pulley at the edge of the table and the mass m_1 is hanging freely. Then the tension T of the string is given by
   (A) $T = \frac{(m_1 - m_2)\,g}{m_1 + m_2}$
   (B) $T = \frac{m_1 m_2\,g}{m_1 + m_2}$
   (C) $T = \frac{m_1\,g}{m_2\,(m_1 + m_2)}$
   (D) $T = \frac{m_1\,g}{m_1 + m_2}$ ( )

3. Two particles of unequal masses are connected by a light inextensible string passing over a smooth pulley, and allowed to hang freely. The tension of the string is equal in magnitude to
   (A) the arithmetic mean of the masses of the particles
   (B) harmonic mean of the masses of the particles
   (C) the arithmetic mean of the weights of the particles
   (D) the harmonic mean of the weights of the particles ( )

4. Two weights of 1 kg and 3 kg are connected by a string over a smooth pulley. If the weight of the pulley is neglected, then the pressure with which the pulley is pulled down is
   (A) 3 g N
   (B) 4 g N
   (C) 1 g N
   (D)

5. In deducing the formula $f=\frac{(m_1-m_2)g}{m_1+m_2}$ with usual notation in the motion under connected systems, the tension of the string is considered to be

   (A) zero

   (B) equal to g

   (C) constant throughout the string, but not necessarily equal to g

   (D) variable throughout the string ( )

6. Two masses m and 2 m are attached to the ends of a light inextensible string passing over a small smooth fixed pulley. The thrust on the pulley is

   (A) greater than 3 mg

   (B) less than 3 mg, but greater than mg

   (C) equal to 3 mg

   (D) equal to mg ( )

7. A particle of mass m is drawn upon a smooth inclined plane which makes an angle α to the horizontal by a light inextensible string which passes over a smooth pulley at the top of the plane and has a mass M hanging freely at the other end. The system starts from rest. The acceleration of the system is

   (A) $g \cdot \frac{M+m \sin \alpha}{m-M}$

   (B) $g \cdot \frac{M+m \cos \alpha}{m-M}$

   (C) $g \cdot \frac{M-m \sin \alpha}{m+M}$

   (D) $g \cdot \frac{M-m \sin \alpha}{m-M}$ ( )

8. Two masses each of 20 gms are connected by a light string passing over a smooth pulley. The mass which must be taken from one and added to another in order to give the system an acceleration equal

(A) $\frac{10}{3}$ gms

(B) $\frac{20}{3}$ gms

(C) $\frac{40}{3}$ gms

(D) 10 gms ( )

9. Two equal masses each equal to m are connected by a string and hang over a pulley. If a mass m_1 ($< m$) taken from one side of the pulley is added to the mass on the other side, then the pressure on the pulley

(A) is decreased, but does not vanish

(B) is increased

(C) remains constant

(D) vanishes ( )

10. Masses 3 M and 5 M are connected by a string passing over a smooth pulley. At the end of 8 s, a mass 2 M is picked up by the ascending body. After this

(A) the acceleration of the system is $\frac{2}{5}$ times the original acceleration

(B) the acceleration of the system is $\frac{3}{7}$ times the original acceleration

(C) the system stops

(D) the system moves with uniform velocity ( )

11. Two masses of 1 kg each are connected by a string One mass is placed on a smooth horizontal table and the other is suspended freely in air, after passing over a pulley placed at the edge of the table. If the string is stretched throughout the journey, the force

(A) equal to its weight

(B) $\sqrt{2}$ times its weight

(C) half of its weight

(D) none of the above ( )

12 A mass of 1 kg placed on a smooth horizontal table which is 3 m long is tied with a light inextensible string parallel to the edge of the table. The string passes over a small smooth pulley fixed at an edge of the table and carries a vertically hanging mass of 100 g. Initially, the mass on the table is at a distance 2.5 m from the pulley. After $\frac{1}{100}$th of a second, the hanging mass suddenly falls off the string. Then the mass on the table

(A) moves forward, but does not leave the table

(B) moves backward, but does not leave the table

(C) moves forward and leaves the table

(D) moves backward and leaves the table ( )

13 A string passes across a smooth table at right angles to opposite edges. Two masses, P and Q ($P > Q$), hanging vertically are attached to the string at its two ends. If a mass M be attached to the portion of the string lying on the table, the acceleration of the system when left to itself will be

(A) $\frac{(P-Q)g}{P+Q+M}$

(B) $\frac{(P+Q)g}{P+Q+M}$

(C) $\frac{Pg}{P+Q+M}$

(D) $\frac{Qg}{P+Q+M}$ ( )

14. Two particles, A and B, of masses 1 kg and 10 kg respectively are connected by a light inextensible string passing over a light smooth pulley at the edge of a smooth horigontal table, B lying on the table

(A) A can not move

(B) A moves downwards with an acceleration $\frac{g}{11}$ m/s^2

(C) A moves downwards with an acceleration $\frac{g}{9}$ m/s^2

(D) A moves downwards with an acceleration $\frac{9g}{11}$ m/s^2 ( )

15. Masses 2 kg and 6 kg are suspended by a light inextensible string hanging around a pulley of negligible mass and the pulley is placed on a light frictionless bar, the portions of the string on both sides of the bar being vertical. The whole system is now placed on one of the scale pans of a balance. When the motion ensues, the reading of the balance will be

(A) 8 kg

(B) 3 kg

(C) 6 kg

(D) none of the above ( )

16. Four identical blocks each of mass m are joined by strings and placed in a line on a smooth table one after another. If a force F is applied to the extreme right block, the tension of the string between this block and the adjacent block is

(A) F

(B) $\frac{1}{2}F$

(C) $\frac{1}{4}F$

(D) $\frac{3}{4}F$ ( )

17. A light string passing over a light smooth pulley carries masses 3 kg and 5 kg at its ends. The system is allowed to move from rest. If the string breaks after each mass has moved a distance of 9 m, then

(A) 10 m

(B) 12 m

(C) 2.5 m

(D) 2.25 m ( )

18 One end of a string passing over a pulley carries a load w N while from the other end, a monkey climbs up the string with a constant acceleration rising 9.8 m in 1 s keeping the load w N at rest. Then the weight of the monkey is

(A) w N

(B) $\frac{w}{3}$ N

(C) $2w$ N

(D) $3w$ N ( )

19 Two unequal weights are contained in scale pans connected by a string passing over a smooth pulley. If the weights of the pans be negligible, then the pressure between each pan and the contained weight is

(A) greater than twice the tension of the string

(B) equal to the tension of the string

(C) less than the tension of the string

(D) twice the tension of the string ( )

20 Two unequal masses connected by a string hang over a pulley. If the sum of the masses be constant and if f be the acceleration of the system and T, the tension of the string, then

(A) f increases with the increase in T

(B) f decreases with the increase in T

(C) $f + T$ is independent of masses

(D) the variation of f does not depend on T

21. A mass of 3 kg descending vertically draws up a mass of 2 kg by means of a light string passing over a pulley. At the end of 5 s, the

(A) $\frac{5g}{2}$ m upwards

(B) $\frac{5g}{2}$ m downwards

(C) $\frac{g}{2}$ m downwards

(D) $\frac{g}{2}$ m upwards ( )

22. Two masses 10 kg and 5 kg are connected by a string. The smaller mass is placed on a smooth plane inclined at an angle of 30° to the horizon and the string, after passing over a small smooth pulley at the top of the plane, supports the 10 kg mass, which hangs vertically. A force of 20 N is applied down the plane on the smaller mass so that it begins to move down the plane. If $g = 10$ m/s², then the acceleration of the system down the plane is

(A) $\frac{11}{30} g$ m/s²

(B) $-\frac{11}{30} g$ m/s²

(C) $\left(20 - \frac{11}{30}\right) g$ m/s²

(D) $\left(20 + \frac{11}{30}\right) g$ m/s² ( )

23. A mass m_1 is attached to one end of a light inextensible string of tension T, which passes over a smooth fixed pulley and to the other end is attached a smooth pulley of mass m_2 over which passes another string of tension T_1 with mass m_3 and m_4 at its ends. If the mass m_1 moves downwards, then the equation of motion of the mass m_2 is

(A) $m_2 f = T - 2T_1$

(B) $m_2 f = T - m_2 g$

(C) $m_2 f = T - m_2 g - 2T_1$

24. A mass m is drawn up a smooth inclined plane of height h and length l by means of a string passing over the top of the plane, from the other end of which hangs a mass m'. If m starts from the bottom and just reaches the top of the plane, m' must be detached after m has moved through a distance

(A) $\frac{m+m}{m'} \cdot \frac{hl}{h+l}$

(B) $\frac{m}{m+m'} \cdot \frac{hl}{h+l}$

(C) $\frac{m+m'}{m} \cdot \frac{2hl}{h+l}$

(D) $\frac{m}{m+2m} \cdot \frac{hl}{h+l}$ ( )

25 Two smooth inclined planes of equal heights, whose inclinations to the horizon are 30° and 60° respectively are placed back to back. Two bodies of masses 6 g and 10 g placed on them respectively are connected by a light inextensible string which passes over a smooth pulley at the common vertex of the planes. Then the acceleration of the system is

(A) $\frac{g}{4}(5\sqrt{3}+3)$ m/s^2

(B) $\frac{g}{16}(5\sqrt{3}-3)$ m/s^3

(C) $\frac{g}{16}(5-3\sqrt{3})$ m/s^2

(D) $\frac{g}{4}(5+3\sqrt{3})$ m/s^2 ( )

ANSWERS

1. (B) 2. (B) 3. (D) 4. (A) 5. (C) 6. (B) 7. (C) 8. (B) 9. (A) 10. (D) 11. (C) 12. (C) 13. (A) 14. (B) 15. (C) 16. (D) 17. (D) 18. (B) 19. (B) 20. (B) 21. (D) 22. (B) 23. (C) 24. (A) 25. (B)

VII. Topic . Projectiles

1 For a given horizontal range with a given velocity of projection, the number of the angles of projection will be

(A) one

(B) two

(C) three

(D) more than three ( )

2 A projectile thrown at an angle of 25° has a certain range. The other angle of projection having the same range for the same velocity of projection will be

(A) 20°

(B) 35°

(C) 65°

(D) 50 ( )

3 A body is projected with a velocity u at an angle α to the horizon At any instant t after start, the horizontal component of the acceleration of the body will be

(A) gt

(B) $-g$

(C) 0

(D) g

4. A body is projected with a velocity u at an angle α to the horizon. At any instant t after start, the vertical displacement of the body will be

(A) $u \cos \alpha \, . \, t$

(B) $u \sin \alpha \, . \, t$

(C) $u \sin \alpha \, . \, t - \frac{1}{2} g t^2$

5 A particle is projected with a velocity u at an angle α in a vacuum After time t, if $P(x, y)$ be the position of the particle with reference to the co-ordinate axes through the point of projection, then its path is given by the equation

(A) $y = u \sin \alpha - gt$

(B) $y = u \sin \alpha \; t - \frac{1}{2} gt^2$

(C) $y = x \tan \alpha - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \alpha}$

(D) $y = x \tan \alpha - \frac{g}{2} \frac{x^2}{u^2} \cos^2 \alpha$ ( )

6 Masses of four different bodies P, Q, R, S are 1 kg, 2 kg, 3 kg and 4 kg respectively. P, Q, R, S are projected horizontally from a height of 40 m above the ground with respective speeds of 4 m/s, 2 m/s, 3 m/s and 1 m/s. Then

(A) S will reach the ground first

(B) P will reach the ground first

(C) Q and R will reach the ground first

(D) all will reach the ground at the same time ( )

7. A body P is dropped from a height h while another body Q si projected in the horizontal direction from the same height with a speed u. The former takes t_1 s to strike the ground while the latter takes t_2 s Then

(A) $t_1 = t_2$ for all values of u

(B) $t_1 > t_2$ for all values of u

(C) $t_1 < t_2$ for all values of u

(D) $t_1 = t_2$ only when $u = 0$

8 The path of a competitor in a broad jump will be

(A) an arc of a circle

(B) a part of a hyperbola

(C) a parabola

9. With a given velocity of projection in all possible directions in a vertical plane, the locus of the foci of the parabolas is

   (A) a parabola

   (B) a circle

   (C) a straight line

   (D) an ellipse ( )

10. With a given velocity of projection in all possible directions in a vertical plane, the locus of the vertices of all parabolas is

   (A) a parabola

   (B) a circle

   (C) an ellipse

   (D) none of the above ( )

11. A train is moving with a uniform velocity of 72 km/h. A stone is thrown vertically upwards inside the train with a velocity of 20 m/s and it returns to the floor of the train. A passenger in the train will see that the stone describes

   (A) a straight vertical path

   (B) a parabolic path

   (C) an elliptic path

   (D) none of the above paths ( )

12. A body is projected upwards with a given velocity making an angle α with the horizon. When $\alpha = \frac{\pi}{2}$, the path of the projectile in vacuo is

   (A) a straight line

   (B) a parabola

   (C) one part of a hyperbola

   (D) none of the above

13. A particle is thrown upwards with a certain velocity at an angle α with the horizon. Then the height of the

(A) tan α

(B) sin α

(C) cos α

(D) none of the above ( )

14 A particle is projected with a given velocity at an angle α with the horizon. With reference to the origin of co-ordinates at the point of projection, the only element of the path of the particle, which is independent of α, is

(A) the length of the latus rectum of the parabola

(B) the height of the directrix of the parabola from the origin

(C) the distance of the vertex of the parabola from the origin

(D) none of the above ( )

15 A gum can fire shots with velocity of magnitude u in all directions from a given position on a horizontal plane. All the shots will fall on the plane within a circle of minimum area

(A) $\frac{\pi u^4}{g^2}$

(B) $\frac{\pi u^4}{4g^2}$

(C) $\frac{\pi u^2}{g^2}$

(D) $\frac{\pi u^2}{4g^2}$ ( )

16. Particles are projected from a given point simultaneously with a velocity of the same magnitude u in different directions in a vertical plane. At a particular instant t after projection, they will all be

(A) on a parabola

(B) on a circle of minimum radius ut

(C) on a circle of minimum radius $\frac{u}{t}$

(D) on a circle of minimum radius $\frac{u^2}{}$

17. A body is projected from the top of a tower, 19.6 m high, with a horizontal velocity 50 m/s. The horizontal displacement of the body in the ground will be

(A) 80.4 m

(B) 119.6 m

(C) 100 m

(D) 19.6 m ( )

18. A particle is projected with a velocity v so that the range on a horizontal plane is twice the greatest height attained. Then the range will be

(A) $\frac{16}{65} \cdot \frac{v^2}{g}$

(B) $\frac{4}{5} \cdot \frac{v^2}{g}$

(C) $\frac{v^2}{4g}$

(D) none of the above ( )

19. A man can throw a stone over a maximum distance of 40 m The stone will rise to a maximum height of

(A) 10 m

(B) 20 m

(C) 40 m

(D) 80 m ( )

20. Four balls B_1, B_2, B_3 and B_4 are projected with the same velocities at angles 10°, 25°, 30° and 60° respectively. The maximum horizontal distance will be covered by the ball

(A) B_1

(B) B_2

(C) B_3

(D) B

21 The greatest height to which a man can throw a stone vertically upwards is h. Then the greatest horizontal distance to which he can throw the same stone is

(A) $\frac{h}{2}$

(B) h

(C) $2h$

(D) $4h$ ( )

22. The greatest horizontal distance to which a man can throw with a given initial velocity is h. The greatest height to which he can throw the same stone vertically upwards with the same initial velocity is

(A) $\frac{h}{2}$

(B) h

(C) 2 h

(D) $4h$ ( )

23. The trajectory of a projectile thrown with a given initial velocity gives the maximum horizontal range, R. The greatest height attained by the particle projected with the same initial velocity will be

(A) $\frac{R}{2}$

(B) $\frac{R}{3}$

(C) $\frac{R}{4}$

(D) $\frac{3R}{4}$ ( )

24. A particle is projected with a velocity of v m/s from a point on the ground so as to cover a maximum horizontal range. Then the

(A) $\frac{v^2}{2g}$ m

(B) $\frac{v^2}{g}$ m

(C) $\frac{v^2}{4g}$ m

(D) 0 m ( )

25. If h_1 and h_2 be the greatest heights in two paths of a projectile for a given range R and a given velocity of projection u, then R is equal to

(A) $\sqrt{h_1 h_2}$

(B) $2 \sqrt{h_1 h_2}$

(C) $4 \sqrt{h_1 h_2}$

(D) none of the above ( )

26 The horizontal range and the time of flight of a projectile projected with a velocity u at an angle α to the horizon are respectively R and T. Then the statement which is *not* true is

(A) $2 R \tan \alpha = g T^2$

(B) $2 u^2 = g^2T^2 + 2 u^2 \cos 2 \alpha$

(C) $RTg^2 = v^2 (\cos \alpha - \cos 3 \alpha)$

(D) $Rg \sec \alpha \operatorname{cosec} \alpha = 2 u^2$ ( )

27. For a projectile, if H is the greatest height, T, the time of flight and V, the vertical component of the initial velocity of the projectile projected at an angle α with the horizon, then the relation between them is

(A) $V = HT$

(B) $V = H \sin \alpha \,.\, T$

(C) $\frac{H}{T} = V$

28. Two bodies are projected from the same point with the same initial velocity making angles $\frac{\pi}{4} + \theta$ and $\frac{\pi}{4} - \theta$ respectively with the horizontal. The horizontal ranges of the projectiles are R_1 and R_2 respectively. If R be the maximum horizontal range, then R_1 and R_2 are connected by the relation

(A) $R_1 - R_2 = \frac{R}{2}$

(B) $R_2 - R_1 = \frac{R}{2}$

(C) $R_1 - R_2 = 0$

(D) $R_1 - R_2 = R$ ( )

29 A particle is projected with a velocity u at an angle α with the horizontal. If R be the horizontal range and T be the time of flight, then α is given by

(A) $\tan \alpha = \frac{gT^2}{2R}$

(B) $\cot \alpha = \frac{gT^2}{2R}$

(C) $\tan \alpha = \frac{R}{gT^2}$

(D) $\cot \alpha = \frac{R}{gT^2}$ ( )

30. A shot is seen to pass horizontally just over a vertical pole $\sqrt{3}\, x$ m high and $2\, x$ m away from the point of projection. The angle of projection is

(A) 30°

(B) 60°

(C) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(D) $\tan^{-1}(2\sqrt{3})$ ( )

31. A footballer kicks a ball which just crosses a wall of height 4 m horizontally. If the wall is 8 m away from the player, the velocity

(A) $4\sqrt{g}$ m/s

(B) $4\sqrt{2g}$ m/s

(C) $\sqrt{10g}$ m/s

(D) $4\sqrt{5g}$ m/s ( )

32 Two masses are projected simultaneously each with velocity u from P and Q making angles of 45° and 135° respectively with PQ which is horizontal. Then they can never collide if

(A) $\frac{2u^2}{g} > PQ > \frac{u^2}{g}$

(B) $PQ = \frac{2u^2}{g}$

(C) $PQ < \frac{u^2}{g}$

(D) $PQ > \frac{2u^2}{g}$ ( )

33. A cricket ball thrown from a height of 2 m at an angle of 30° with the horizon is caught by the wicket keeper after 3 s at a height of 50 cm from the ground. The velocity with which the ball is thrown is

(A) 28.4 m/s

(B) 30.4 m/s

(C) 16.5 m/s

(D) 17.7 m/s ( )

34 A stone is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. The distance of the point where it strikes the ground from the foot of the tower is

(A) h

(B) $\frac{h}{2}$

(C) $2h$

(D) $3h$

35 A ball rolls off the top of a stairway, which is marked as the Oth step, with a velocity of 2m/s Each step is 25 m high and ·25 m wide. The ball at first hits the *n*th step after rolling off the Oth step where *n* is equal to

(A) 2

(B) 3

(C) 4

(D) none of the above ( )

36. If t s be the time in which a projectile reaches a point P of its path and t_1 s be the time from P till it strikes the horizontal plane through the point of projection, then the height of the point P from the plane is

(A) $\frac{1}{2} gtt_1$

(B) $\frac{1}{2} gt\,(2t_1+t)$

(C) $gt\,(t_1 - \frac{1}{2}\,t)$

(D) $\frac{1}{2} gt\,(t_1-t)$ ( )

37. Two equal particles are projected, one vertically upwards and the other at an angle of 45° with the vertical. The greatest height attained by the first particle is equal to

(A) the maximum horizontal range of the second particle

(B) twice the maximum horizontal range of the second particle

(C) half the maximum horizontal range of the second particle

(D) one-third of the maximum horizontal range of the second particle ( )

38. The angle of projection for which the range on the horizontal plane will be half of its maximum range will be

(A) either 15° or $22\frac{1}{2}$°

(B) either 15° or 60°

(C) either 15° or 75°

39. A man stands at a distance $\frac{u^2}{2g}$ from his friend and his friend throws a stone with a velocity u at an angle α to the horizon The man is unsafe if

(A) $0^\circ < \alpha < 15^\circ$

(B) $15^\circ < \alpha < 30^\circ$

(C) $30^\circ < \alpha < 45^\circ$

(D) α has some value or values other than above ( )

40. The line joining two points, A and B, is inclined at an angle α to the horizon. The ratio of the least velocity V to shoot from A to B to the least velocity V' required to shoot from B to A is

(A) $\tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$

(B) $\sec\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$

(C) $\tan\left[\frac{\pi}{4} \quad \frac{\alpha}{2}\right]$

(D) $\tan\left[\frac{\pi}{4} + \frac{\alpha}{2}\right]$ ( )

41. For a given velocity or projection, the ratio of the maximum range down a plane of inclination α to that up the plane is

(A) $\frac{1-\sin\alpha}{1+\sin\alpha}$

(B) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(C) $\frac{1+\sin\alpha}{1-\cos\alpha}$

(D) $\frac{1+\sin\alpha}{1-\sin\alpha}$

ANSWERS

1. (B) 2. (C) 3. (C) 4. (C) 5. (C) 6. (D) 7. (A) 8. (C) 9. (B) 10. (C) 11. (A) 12. (A) 13. (D) 14. (B) 15. (A) 16. (B) 17. (C) 18. (B) 19. (A) 20. (C) 21. (C) 22. (A) 23. (C) 24. (C) 25. (C) 26. (C) 27. (D) 28. (C) 29. (A) 30. (B) 31. (A) 32. (D) 33. (A) 34. (C) 35. (C) 36. (A) 37. (C) 38. (C) 39. (D) 40. (D) 41. (D)

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